

Small Signal Analysis

CM

SMALL SIGNAL MODEL OF THE BJT

- A small signal model is a linear model which is independent of signal amplitude. It may or may not have time dependence

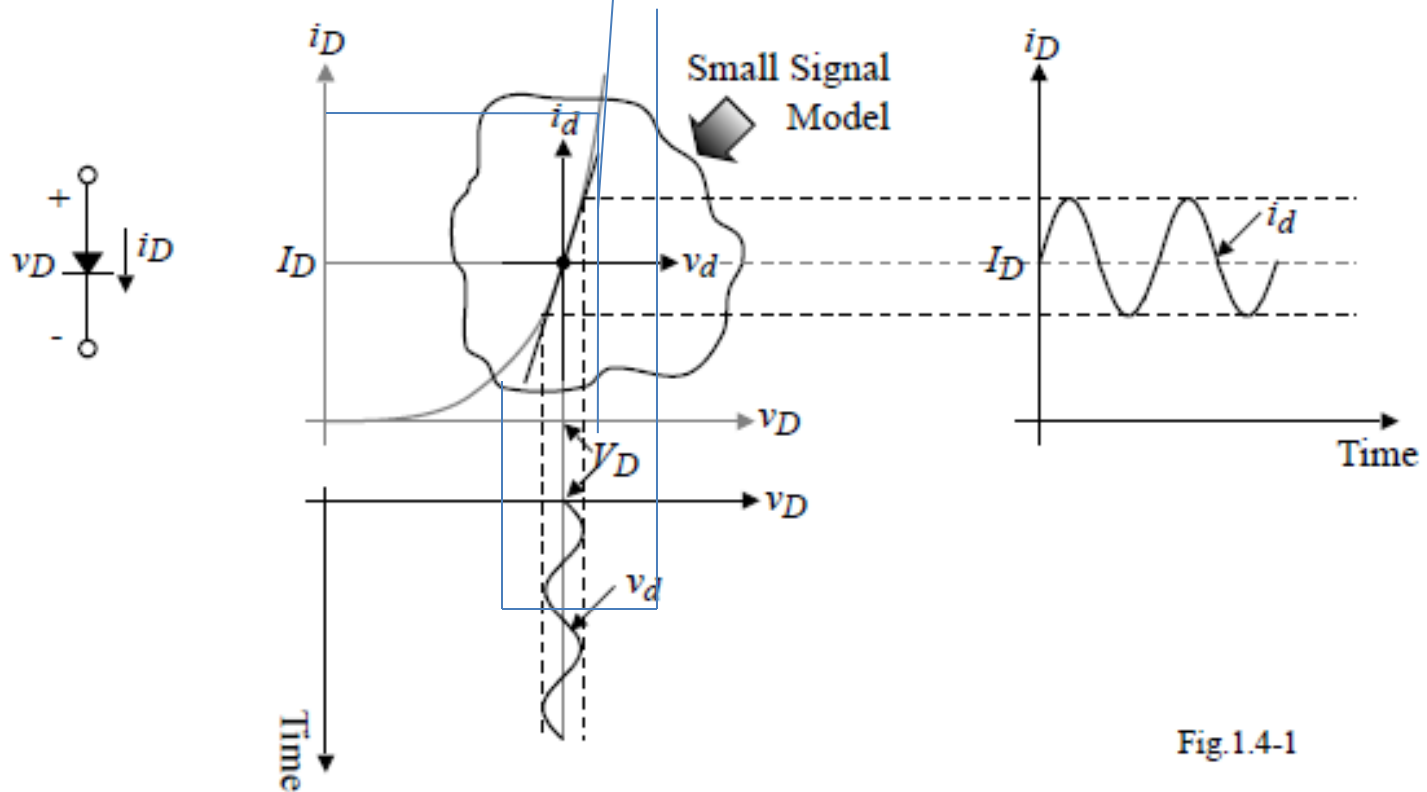
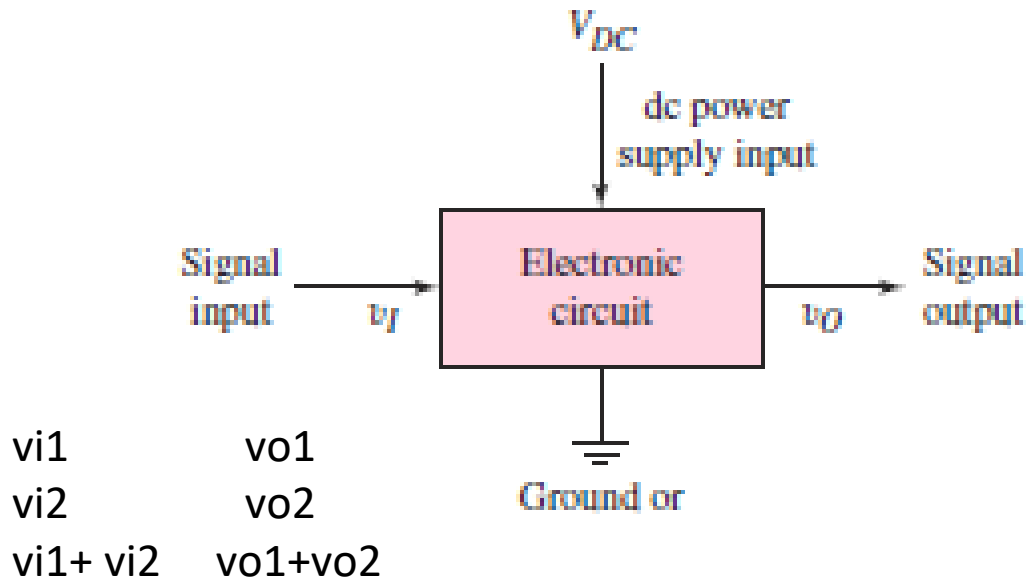
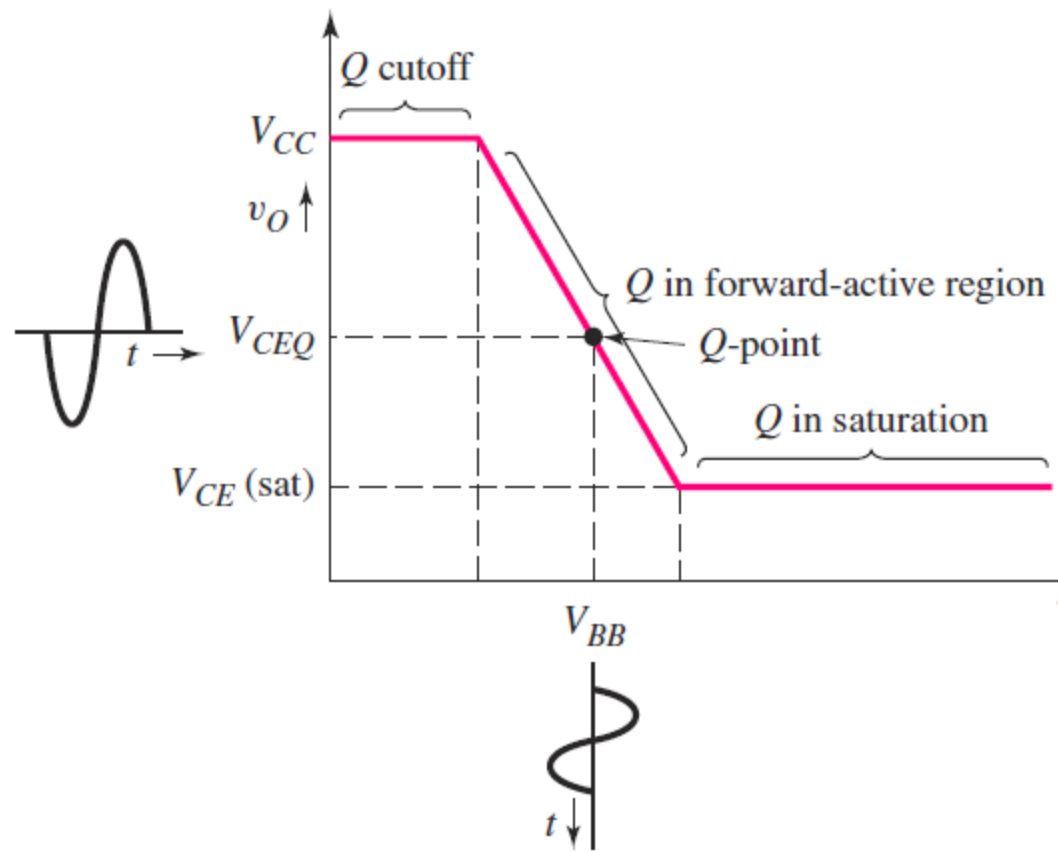
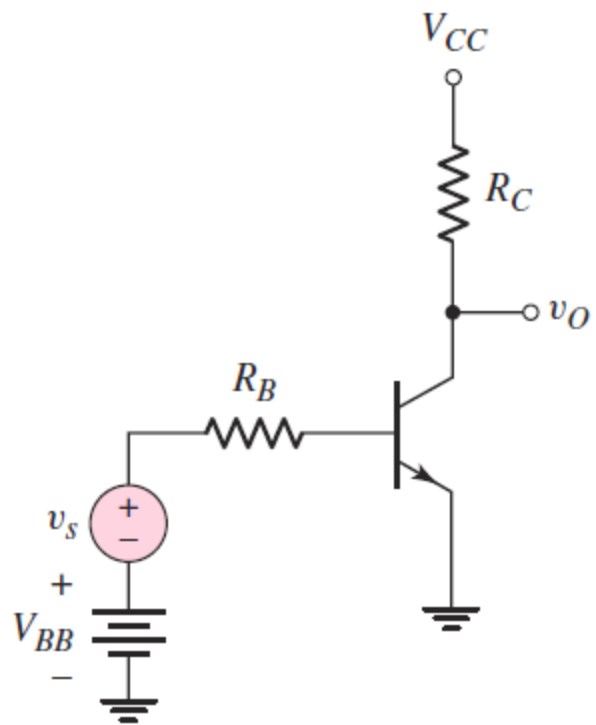


Fig.1.4-1



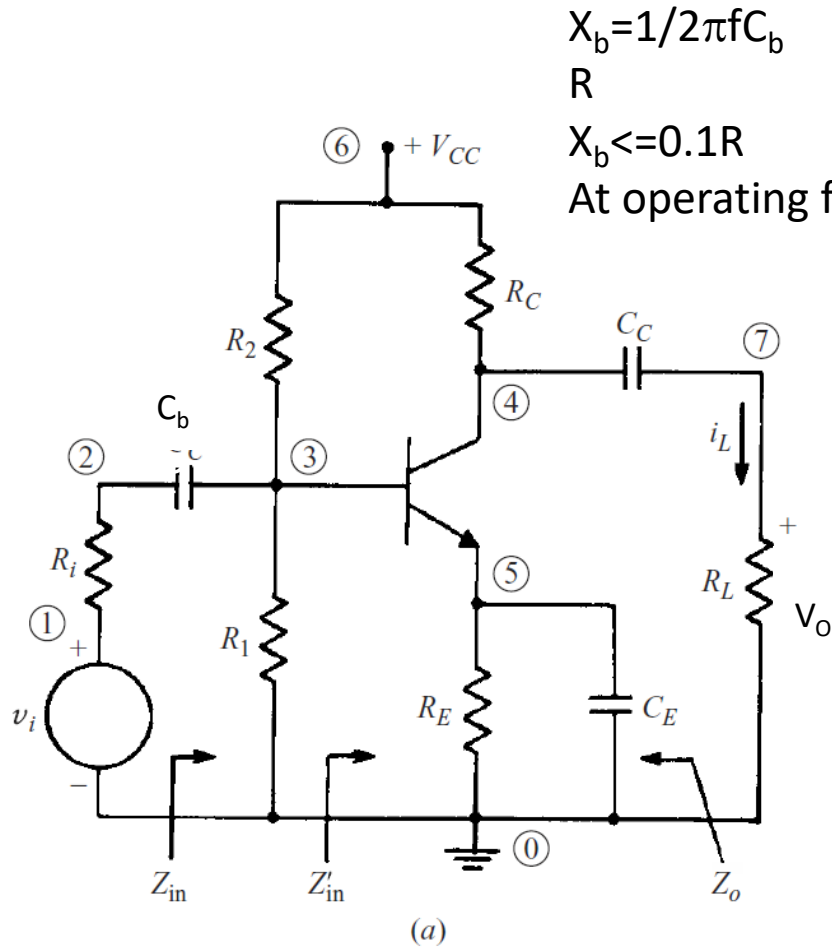
A linear amplifier means that the superposition principle applies. The principle of superposition states: *The response of a linear circuit excited by multiple independent input signals is the sum of the responses of the circuit to each of the input signals alone.*



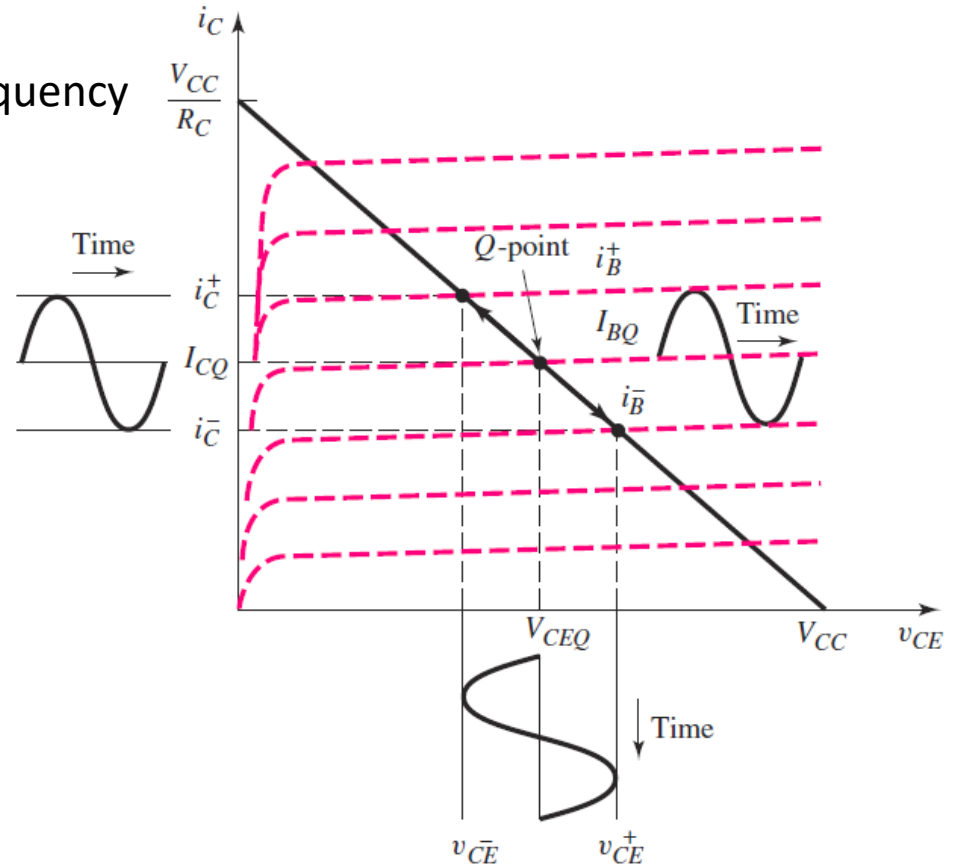
IEEE NOTATION

Variable	Meaning
i_B, v_{BE}	Total instantaneous values
I_B, V_{BE}	DC values
i_b, v_{be}	Instantaneous ac values
I_b, V_{be}	Phasor values

DESIGN OF AMPLIFIER CIRCUIT



$$Z = \sqrt{R^2 + X_b^2} = R$$



Coupling: direct , capacitive and transformer coupling

$$i_B = I_{BQ} + i_b$$

$$i_C = I_{CQ} + i_c$$

$$v_{CE} = V_{CEQ} + v_{ce}$$

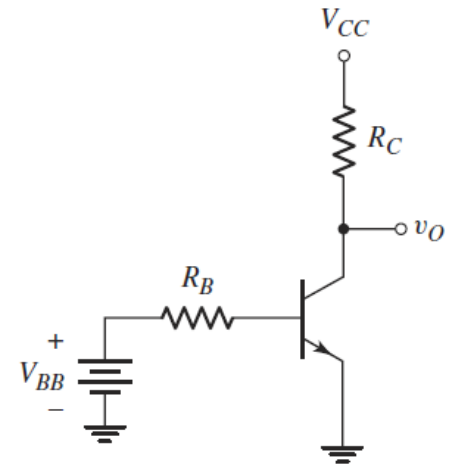
$$V_{BB} = I_{BQ}R_B + V_{BEQ}$$

$$V_{BB} + v_s = i_B R_B + v_{BE}$$

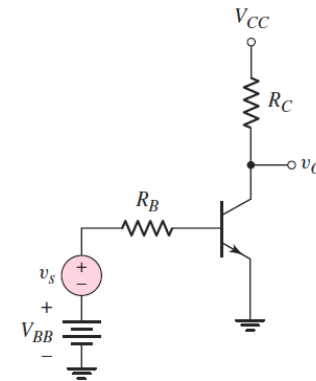
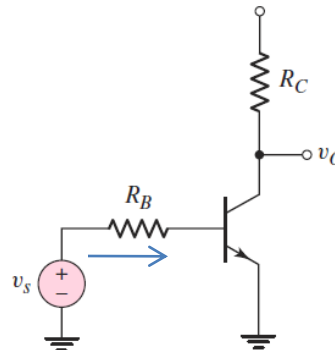
$$V_{BB} + v_s = (I_{BQ} + i_b)R_B + (V_{BEQ} + v_{be})$$

$$V_{BB} - I_{BQ}R_B - V_{BEQ} = i_b R_B + v_{be} - v_s$$

$$v_s = i_b R_B + v_{be}$$



$v_s \rightarrow$ input signal

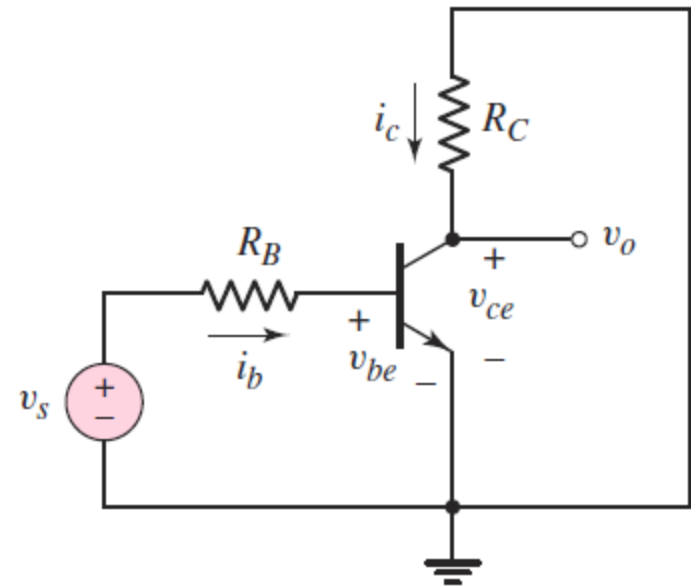
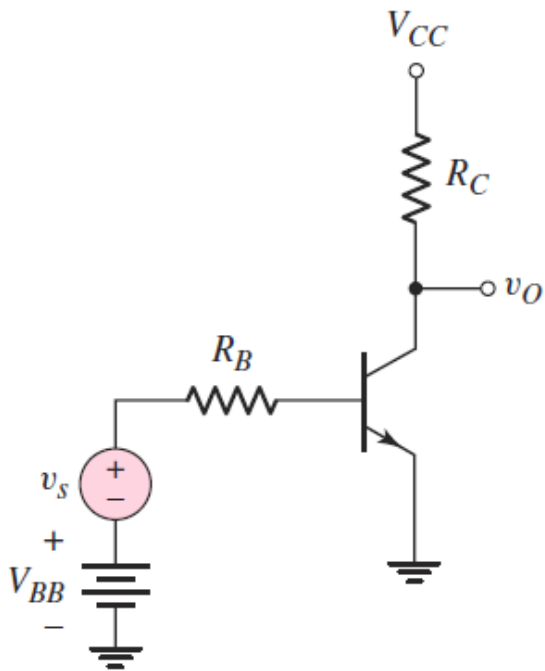


$$V_{CC} = I_{CQ}R_C + V_{CEQ}$$

$$V_{CC} = i_C R_C + v_{CE} = (I_{CQ} + i_c)R_C + (V_{CEQ} + v_{ce})$$

$$i_c R_C + v_{ce} = 0$$

$$v_s = i_b R_B + v_{be}$$



Ac equivalent circuit

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$i_B = \frac{I_S}{\beta} \cdot \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$v_{BE} = V_{BEQ} + v_{be}$$

$$i_B = \frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BEQ} + v_{be}}{V_T}\right) = \frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BEQ}}{V_T}\right) \cdot \exp\left(\frac{v_{be}}{V_T}\right)$$

$$i_B = I_{BQ} \cdot \exp\left(\frac{v_{be}}{V_T}\right)$$

$$i_B \cong I_{BQ} \left(1 + \frac{v_{be}}{V_T}\right) = I_{BQ} + \frac{I_{BQ}}{V_T} \cdot v_{be} = I_{BQ} + i_b$$

$$v_{be} = i_b r_\pi$$

$$v_{be}/V_T \ll 1$$

$$r_{\pi} = \frac{V_T}{I_{BQ}} = \frac{\beta V_T}{I_{CQ}} \quad \text{Input resistance}$$

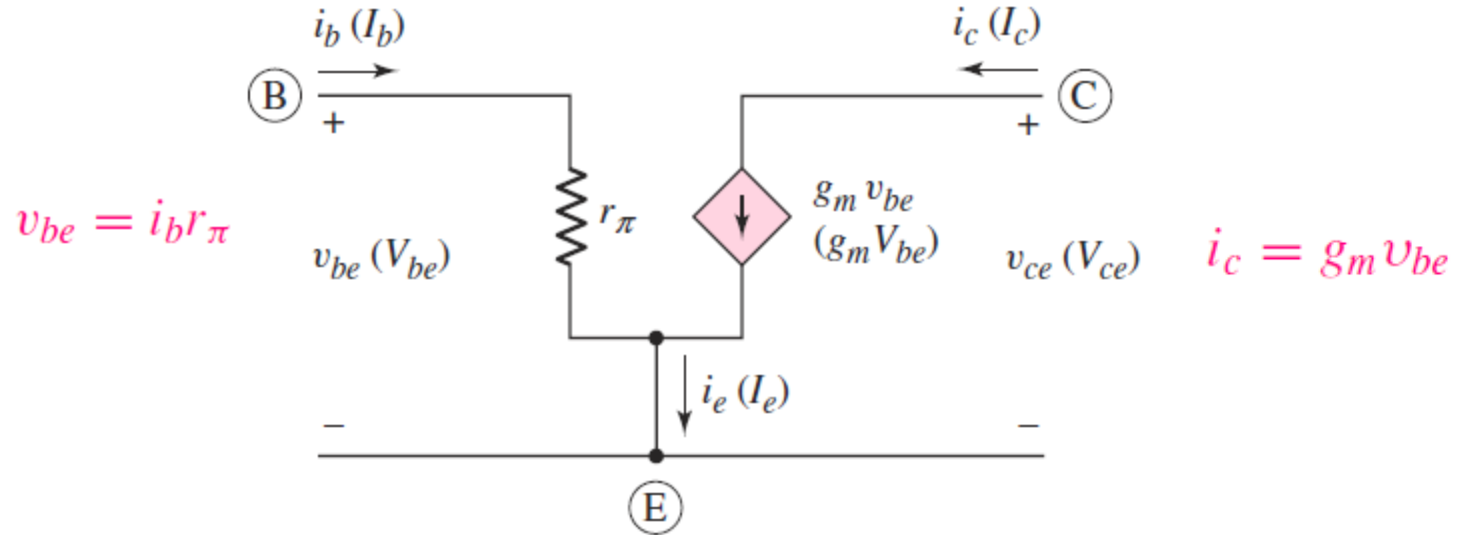
$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$\left. \frac{\partial i_C}{\partial v_{BE}} \right|_{Q\text{-pt}} = \frac{1}{V_T} \cdot I_S \exp\left(\frac{v_{BE}}{V_T}\right) \Big|_{Q\text{-pt}} = \frac{I_{CQ}}{V_T}$$

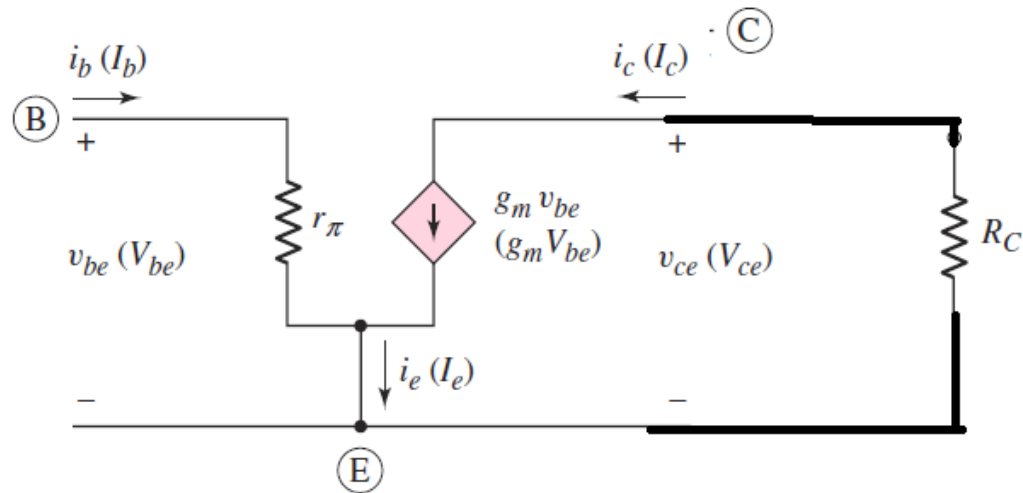
$$g_m = \frac{I_{CQ}}{V_T} \quad \text{transconductance}$$

$$v_{be} = i_b r_{\pi}$$

$$i_c = g_m v_{be}$$



$$i_c R_C + v_{ce} = 0$$

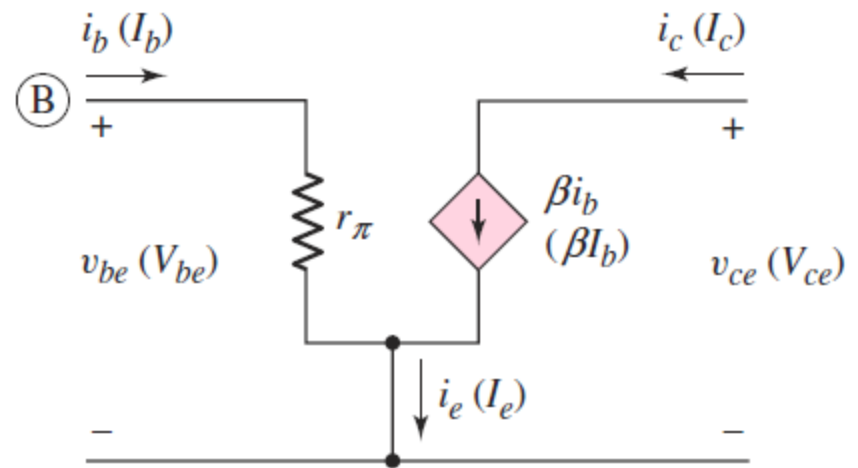


$$\Delta i_C = \left. \frac{\partial i_C}{\partial i_B} \right|_{Q-pt} \cdot \Delta i_B$$

$$i_c = \left. \frac{\partial i_C}{\partial i_B} \right|_{Q-pt} \cdot i_b$$

$$\left. \frac{\partial i_C}{\partial i_B} \right|_{Q-pt} \equiv \beta$$

$$i_c = \beta i_b$$

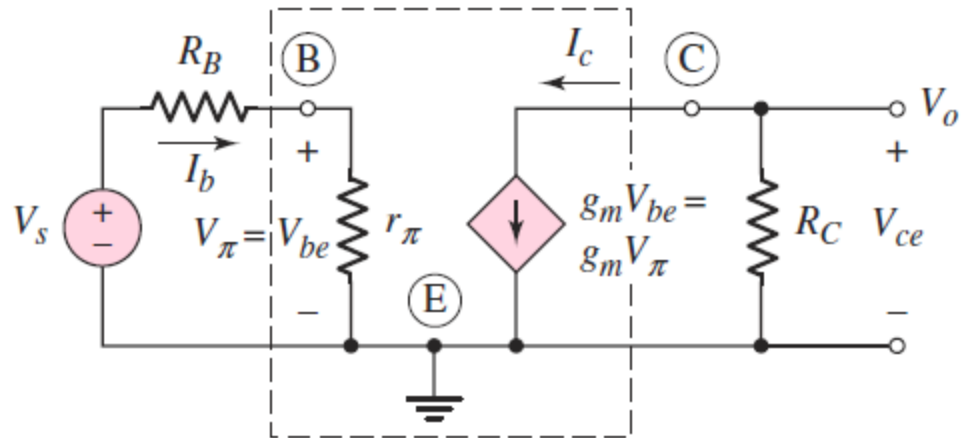
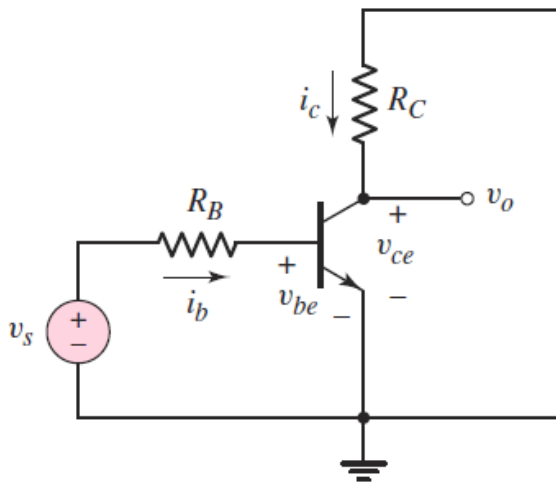


$$r_{\pi} g_m = \left(\frac{\beta V_T}{I_{CQ}} \right) \cdot \left(\frac{I_{CQ}}{V_T} \right) = \beta$$

$$V_o = V_{ce} = -(g_m V_{\pi}) R_C = -\beta / r_{\pi} V_{\pi} R_C$$

Intrinsic voltage gain

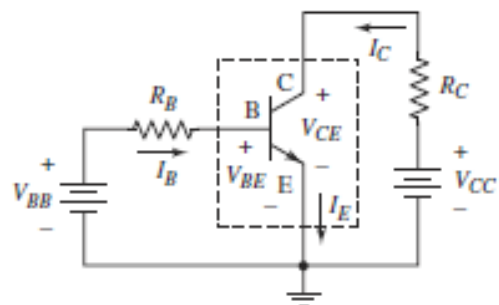
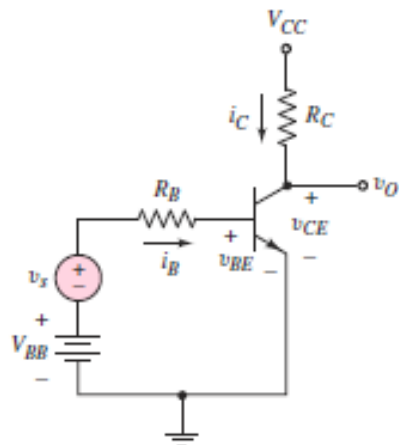
$$A_v = -g_m R_C = -\beta R_C / r_{\pi} = -R_C / (r_e + R_E)$$



$$V_{\pi} = \left(\frac{r_{\pi}}{r_{\pi} + R_B} \right) \cdot V_s$$

$$A_{v_o} = \frac{V_o}{V_s} = -(g_m R_C) \cdot \left(\frac{r_{\pi}}{r_{\pi} + R_B} \right)$$

Assume the transistor and circuit parameters are: $\beta = 100$, $V_{CC} = 12 \text{ V}$, $V_{BE} = 0.7 \text{ V}$, $R_C = 6 \text{ k}\Omega$, $R_B = 50 \text{ k}\Omega$, and $V_{BB} = 1.2 \text{ V}$.



(a) DC equivalent Circuit

gm, re
voltage

$$I_B = \frac{V_{BB} - V_{BE(\text{on})}}{R_B}$$

$$I_C = \beta I_B$$

and

$$V_{CC} = I_C R_C + V_{CE}$$

$$I_{CQ} = 1 \text{ mA and } V_{CEQ} = 6 \text{ V.}$$

AC Solution: The small-signal hybrid- π parameters are

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

and

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1}{0.026} = 38.5 \text{ mA/V}$$

$$A_v = \frac{V_o}{V_s} = -(g_m R_C) \cdot \left(\frac{r_{\pi}}{r_{\pi} + R_B} \right)$$

or

$$= -(38.5)(6) \left(\frac{2.6}{2.6 + 50} \right) = -11.4$$

The circuit parameters for the circuit in Figure _____ are $V_{CC} = 3.3$ V, $V_{BB} = 0.850$ V, $R_B = 180$ k Ω , and $R_C = 15$ k Ω . The transistor parameters are $\beta = 120$ and $V_{BE}(\text{on}) = 0.7$ V. (a) Determine the Q -point values I_{CQ} and V_{CEQ} . (b) Find the small-signal hybrid- π parameters g_m and r_π . (c) Calculate the small-signal voltage gain. (Ans. (a) $I_{CQ} = 0.1$ mA, $V_{CEQ} = 1.8$ V; (b) $g_m = 3.846$ mA/V, $r_\pi = 31.2$ k Ω ; (c) $A_v = -8.52$).








Problem-Solving Technique: Bipolar AC Analysis

Since we are dealing with linear amplifier circuits, superposition applies, which means that we can perform the dc and ac analyses separately. The analysis of the BJT amplifier proceeds as follows:

1. Analyze the circuit with only the dc sources present. This solution is the dc or quiescent solution, which uses the dc signal models for the elements, as listed in Table 2. The transistor must be biased in the forward-active region in order to produce a linear amplifier.
2. Replace each element in the circuit with its small-signal model, as shown in Table 6.2. The small-signal hybrid- π model applies to the transistor although it is not specifically listed in the table.
3. Analyze the small-signal equivalent circuit, setting the dc source components equal to zero, to produce the response of the circuit to the time-varying input signals only.

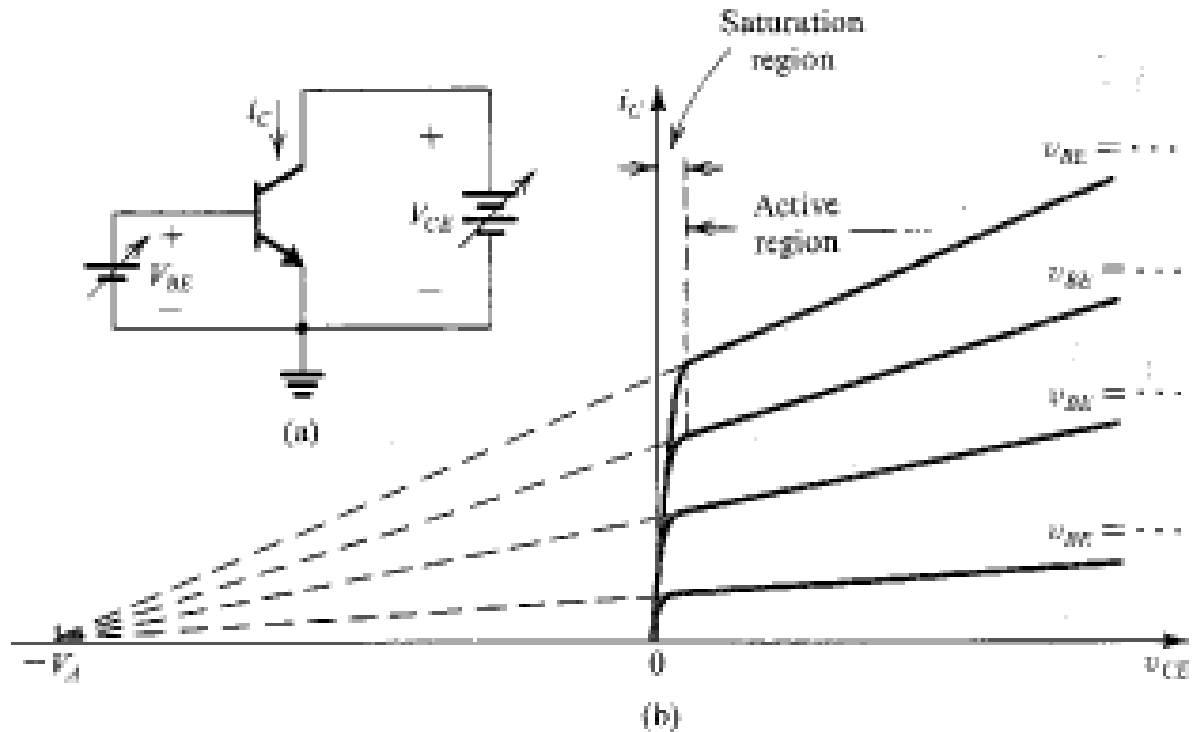
Table 4.2

Transformation of elements in dc and small-signal analysis

Element	I - V relationship	DC model	AC model
Resistor	$I_R = \frac{V}{R}$	R	R
Capacitor	$I_C = sCV$	Open 	C
Inductor	$I_L = \frac{V}{sL}$	Short 	L
Diode	$I_D = I_S(e^{v_D/V_T} - 1)$	$+V_\gamma - r_f$	$r_d = V_T/I_D$ 
Independent voltage source	$V_S = \text{constant}$	$+V_S -$ 	Short 
Independent current source	$I_S = \text{constant}$	I_S 	Open 

Hybrid- π Equivalent Circuit, Including the Early Effect

$$i_C = I_S \left[\exp \left(\frac{v_{BE}}{V_T} \right) \right] \cdot \left(1 + \frac{v_{CE}}{V_A} \right)$$



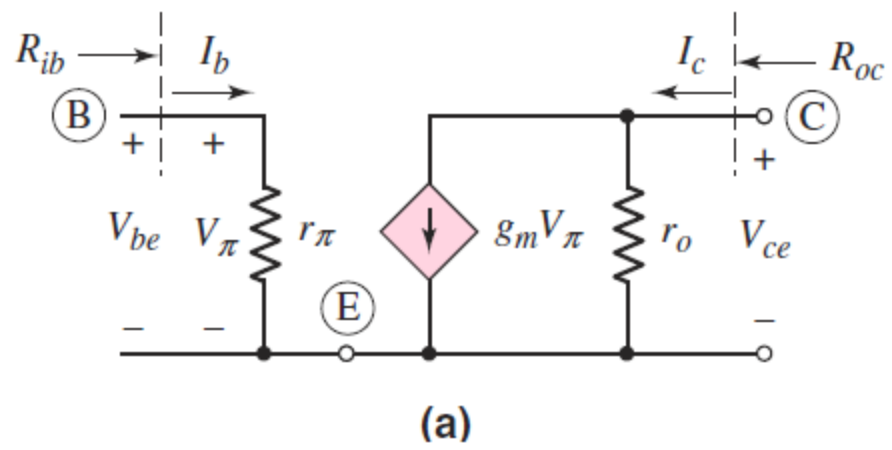
V_A is the Early voltage and is a positive quantity.

The output resistance r_o is defined as

$$r_o = [\partial v_{CE} / \partial i_C]_{Q-pt} = \left[[\partial i_C / \partial v_{CE}]_{Q-pt} \right]^{-1}$$

$$\begin{aligned} \left. \frac{\partial i_C}{\partial v_{CE}} \right|_{Q-pt} &= \frac{\partial}{\partial v_{CE}} \left\{ I_S \left[\exp\left(\frac{v_{BE}}{V_T}\right) \left(1 + \frac{v_{CE}}{V_A}\right) \right] \right\} \Big|_{Q-pt} \\ &\cong \frac{I_{CQ}}{V_A} \end{aligned}$$

$$r_o = \frac{V_A}{I_{CQ}}$$



$$r_o = \frac{V_A}{I_{CQ}} = \frac{50}{1 \text{ mA}} = 50 \text{ k}\Omega$$

$$A_v = \frac{V_o}{V_s} = -g_m(R_C \parallel r_o) \left(\frac{r_\pi}{r_\pi + R_B} \right) = -(38.5)(6 \parallel 50) \left(\frac{2.6}{2.6 + 50} \right) = -10.2$$

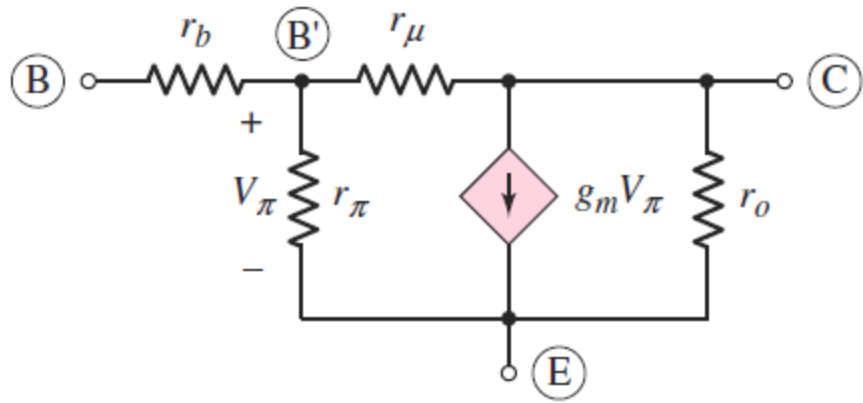
$$\beta = 150,$$

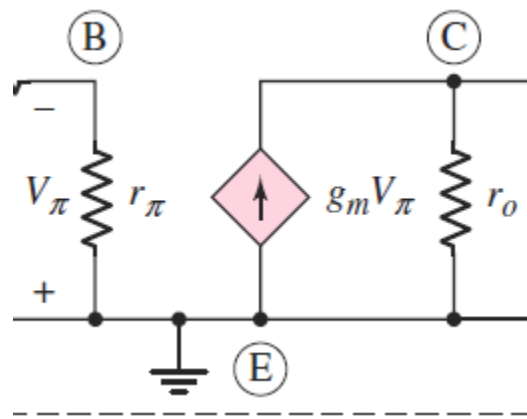
$V_A = 150$ V. The circuit parameters are $V_{CC} = 5$ V,
 $V_{BB} = 1$ V, $R_B = 100$ k, and $R_C = 6$ k.

- (a) Determine the small-signal hybrid- π parameters g_m , r_π , and r_o .
(b) Find the small-signal voltage gain $A_v = V_o/V_s$.

$$g_m = 18.75 \text{ mA/V}, r_\pi = 8 \text{ k}, r_o = 308 \text{ k};$$

$$(b) A_v = -8.17$$



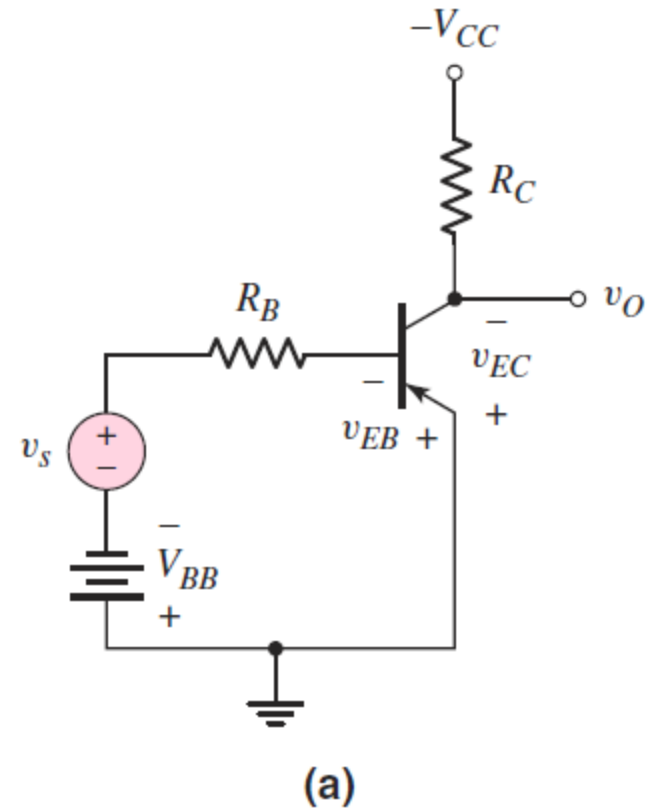


$V_{CC}=5V$
 $R_B= 50k$
 $R_C= 3k$
 $V_{BB}=3.65V$
 $\beta=80$

$$A_v = -4.62$$

$$R_i = R_B + r_\pi = 50 + 2 = 52 \text{ k}\Omega$$

$$R_o = R_C \parallel r_o = 3 \parallel \infty = 3 \text{ k}\Omega$$

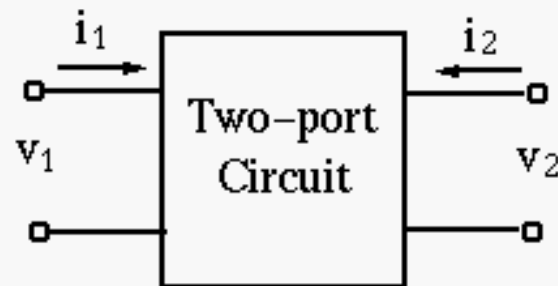


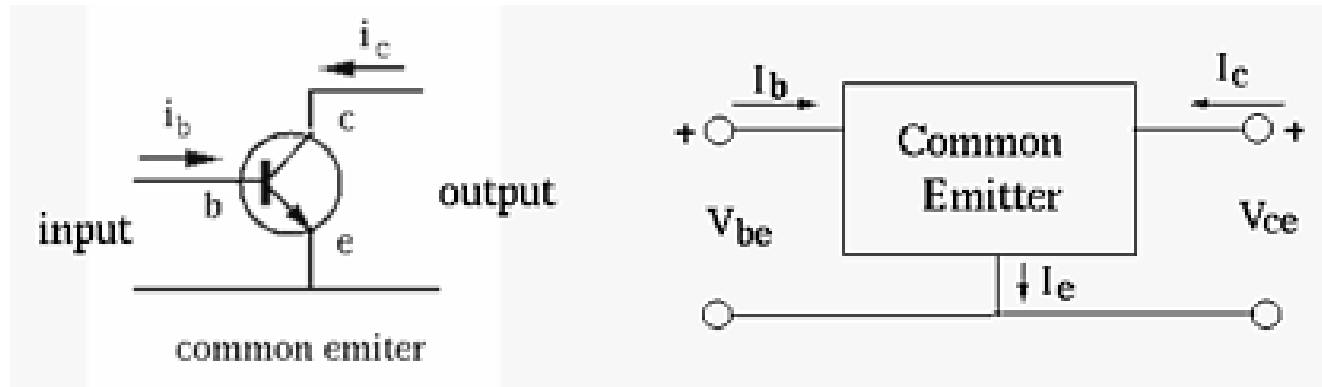
Hybrid parameters

Independent signals: Input current, output voltage
dependent signals: Input voltage, output current

$$V_i = f_1(i_1, V_o) \text{-----(1)}$$

$$I_o = f_2(i_1, V_o) \text{-----(2)}$$





$$V_{be} = h_{ie}I_b + h_{re}V_{ce}$$

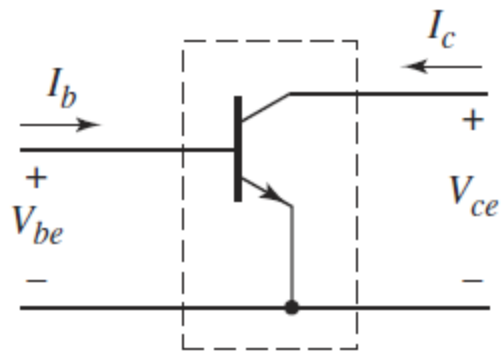
$$I_c = h_{fe}I_b + h_{oe}V_{ce}$$

$h_{ie} = V_{be}/I_b \mid v_{CE} = \text{constant}$ output circuit is short circuited to ac

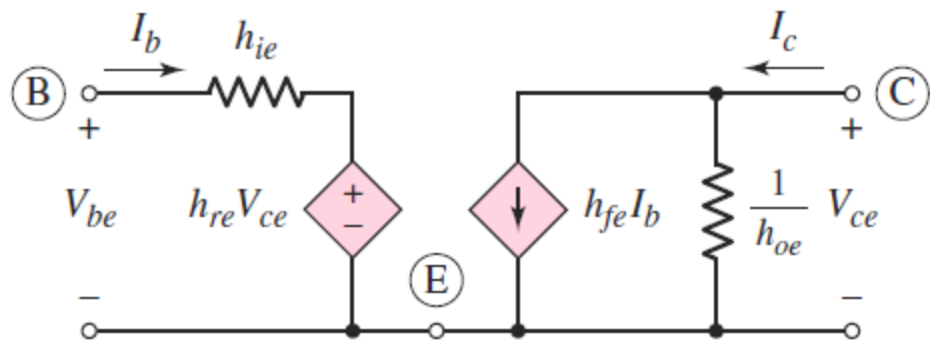
$h_{re} = V_{be}/V_{ce} \mid I_b = 0$ input circuit is open circuited to ac

$h_{fe} = I_c/I_b \mid v_{CE} = \text{constant}$

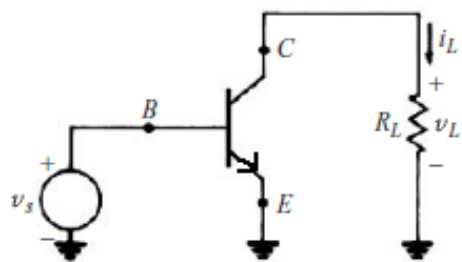
$h_{oe} = I_c/V_{ce} \mid I_b = 0$



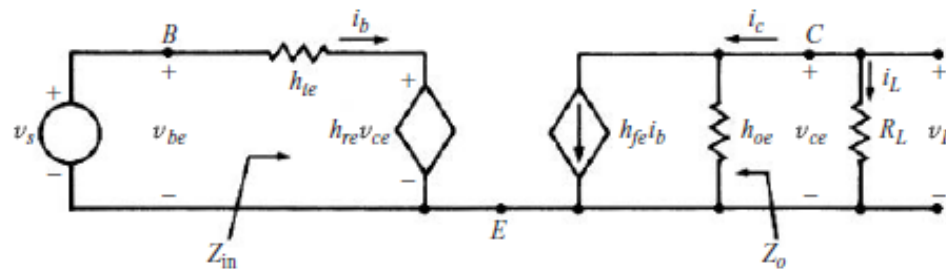
(a)



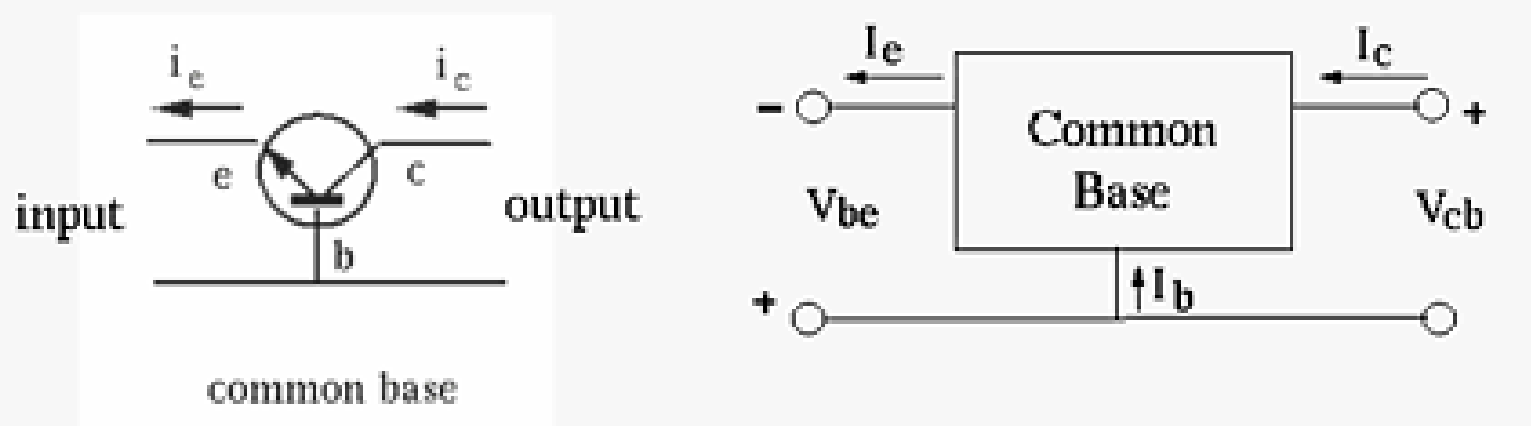
(b)



(a)



(b)

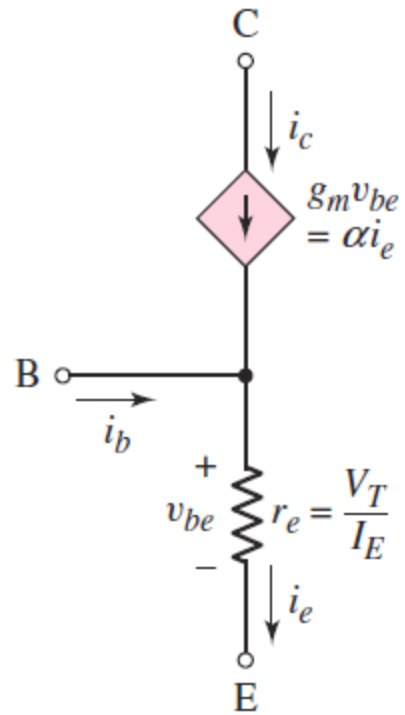


$$h_{ib} = V_{eb} / I_e \quad | \quad v_{CB} = \text{constant} \quad \text{output circuit is short circuited to ac}$$

$$h_{rb} = V_{eb} / V_{cb} \quad | \quad I_e = 0 \quad \text{input circuit is open circuited to ac}$$

$$h_{fb} = I_c / I_e \quad | \quad v_{CB} = \text{constant}$$

$$h_{ob} = I_c / V_{cb} \quad | \quad I_e = 0$$



Common-Base Transistor Connection

$$v_{eb} = h_{ib}i_e + h_{rb}v_{cb}$$

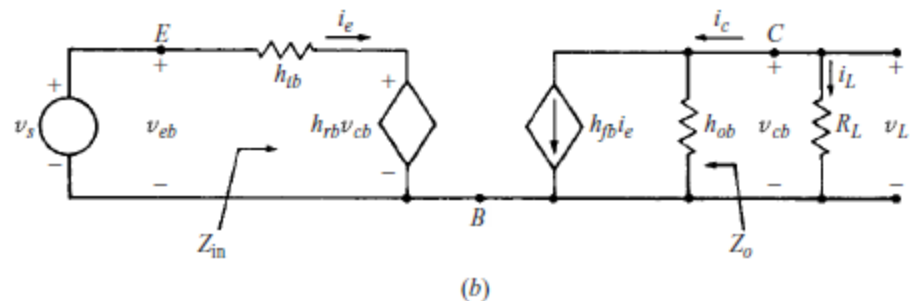
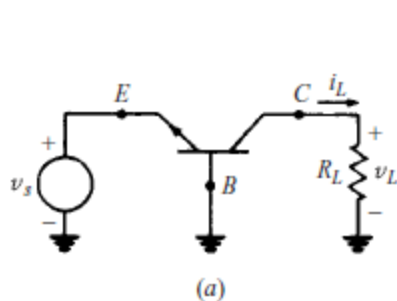
$$i_c = h_{fb}i_e + h_{ob}v_{cb}$$

Input resistance $h_{ib} \equiv \left. \frac{\partial v_{EB}}{\partial i_E} \right|_Q \approx \left. \frac{\Delta v_{EB}}{\Delta i_E} \right|_Q$

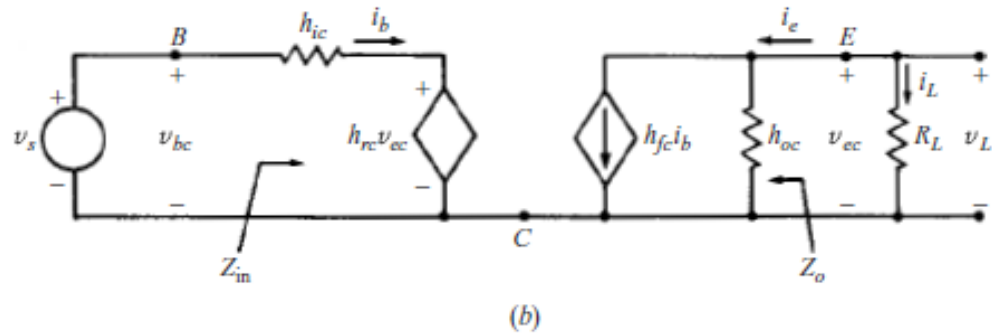
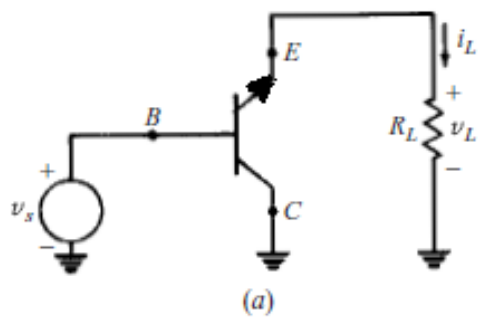
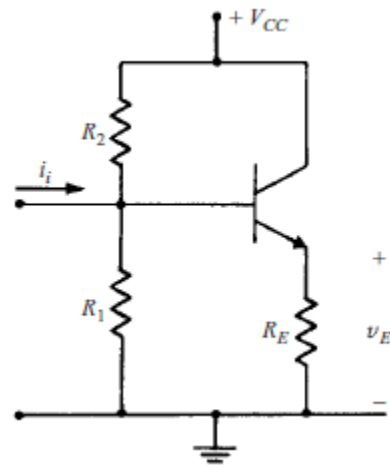
Reverse voltage ratio $h_{rb} \equiv \left. \frac{\partial v_{EB}}{\partial v_{CB}} \right|_Q \approx \left. \frac{\Delta v_{EB}}{\Delta v_{CB}} \right|_Q$

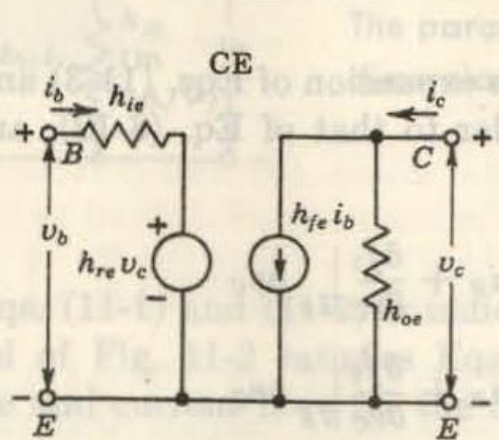
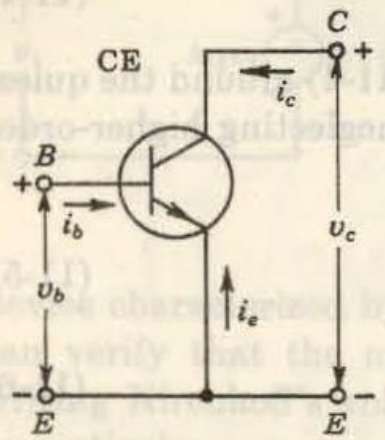
Forward current gain $h_{fb} \equiv \left. \frac{\partial i_C}{\partial i_E} \right|_Q \approx \left. \frac{\Delta i_C}{\Delta i_E} \right|_Q$

Output admittance $h_{ob} \equiv \left. \frac{\partial i_C}{\partial v_{CB}} \right|_Q \approx \left. \frac{\Delta i_C}{\Delta v_{CB}} \right|_Q$



Common-Collector Amplifier

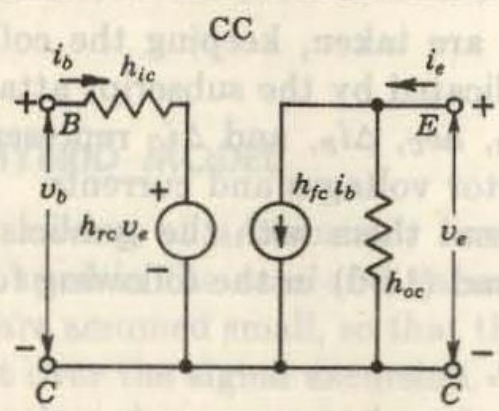
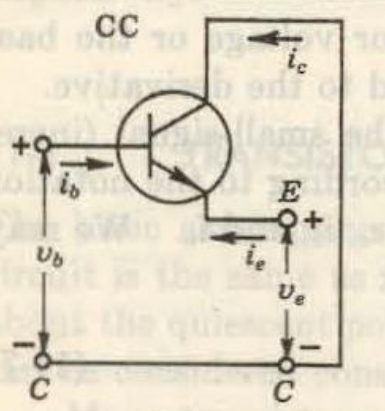




CE

$$v_b = h_{ie} i_b + h_{re} v_c$$

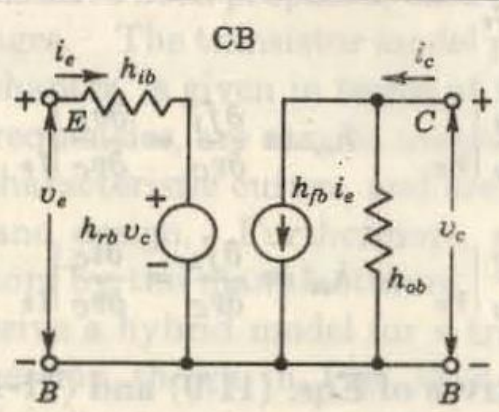
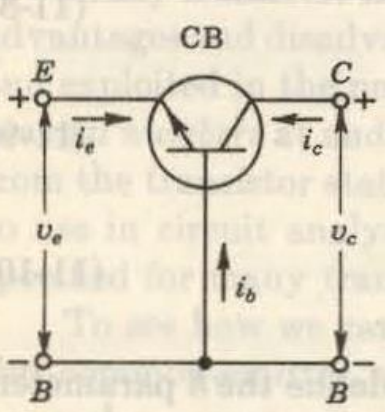
$$i_c = h_{fe} i_b + h_{oe} v_c$$



CC

$$v_b = h_{ic} i_b + h_{rc} v_e$$

$$i_e = h_{fc} i_b + h_{oc} v_e$$



CB

$$v_e = h_{ib} i_e + h_{rb} v_c$$

$$i_c = h_{fb} i_e + h_{ob} v_c$$

CONVERSION OF h-PARAMETERS

reasonable approximations $h_{re} \ll 1$ and $h_{fe} + 1 \gg h_{ie} h_{oe}$

$$v_{be} = h_{ie} i_b + h_{re} v_{ce}$$

$$v_{eb} = -h_{ie} i_b - h_{re} v_{ce}$$

$$i_b = -i_e - i_c = -i_e - h_{fe} i_b - h_{oe} v_{ce}$$

$$-i_b = \frac{1}{h_{fe} + 1} i_e + \frac{h_{oe}}{h_{fe} + 1} v_{ce}$$

$$v_{ce} = v_{cb} - v_{eb}$$

$$v_{eb} = h_{ie} \left[\frac{1}{h_{fe} + 1} i_e + \frac{h_{oe}}{h_{fe} + 1} v_{ce} \right] - h_{re} v_{ce}$$

$$= \frac{h_{ie}}{h_{fe} + 1} i_e + \left[\frac{h_{ie} h_{oe}}{h_{fe} + 1} - h_{re} \right] v_{ce}$$

$$= \frac{h_{ie}}{h_{fe} + 1} i_e + \left[\frac{h_{ie} h_{oe}}{h_{fe} + 1} - h_{re} \right] (v_{cb} - v_{eb})$$

$$v_{eb} \left[1 - \frac{h_{ie} h_{oe}}{h_{fe} + 1} - h_{re} \right] = \frac{h_{ie}}{h_{fe} + 1} i_e + \left[\frac{h_{ie} h_{oe}}{h_{fe} + 1} - h_{re} \right] v_{cb}$$

reasonable approximations $h_{re} \ll 1$ and $h_{fe} + 1 \gg h_{ie} h_{oe}$

$$h_{ib} = \frac{h_{ie}}{h_{fe} + 1}$$

$$h_{rb} = \frac{h_{ie}h_{oe}}{h_{fe} + 1} - h_{re}$$

$$i_c = h_{fe}i_b + h_{oe}v_{ce}$$

$$-i_b = \frac{1}{h_{fe} + 1} i_e + \frac{h_{oe}}{h_{fe} + 1} v_{ce}$$

$$v_{ce} = v_{cb} - v_{eb}$$

$$v_{eb} \approx \frac{h_{ie}}{h_{fe} + 1} i_e + \left(\frac{h_{ie}h_{oe}}{h_{fe} + 1} - h_{re} \right) v_{cb}$$

$$i_c = - \left[\frac{h_{fe}}{h_{fe} + 1} + \frac{h_{oe}h_{ie}}{(h_{fe} + 1)^2} \right] i_e - h_{oe} \left[\frac{h_{ie}h_{oe}}{(h_{fe} + 1)^2} - \frac{h_{re} + 1}{h_{fe} + 1} \right] v_{cb}$$

$$i_c \approx - \frac{h_{fe}}{h_{fe} + 1} i_e + \frac{h_{oe}}{h_{fe} + 1} v_{cb}$$

$$h_{fb} = -\frac{h_{fe}}{h_{fe} + 1}$$

$$h_{ob} = \frac{h_{oe}}{h_{fe} + 1}$$

$$h_{ib} = \frac{h_{ie}}{h_{fe} + 1}$$

$$h_{rb} = \frac{h_{ie}h_{oe}}{h_{fe} + 1} - h_{re}$$

$$h_{fc} \quad | \quad -(1 + h_{fe})$$

$$h_{rc} \quad | \quad 1 - h_{re} \approx 1$$

$$h_{oc} \quad | \quad h_{oe}$$

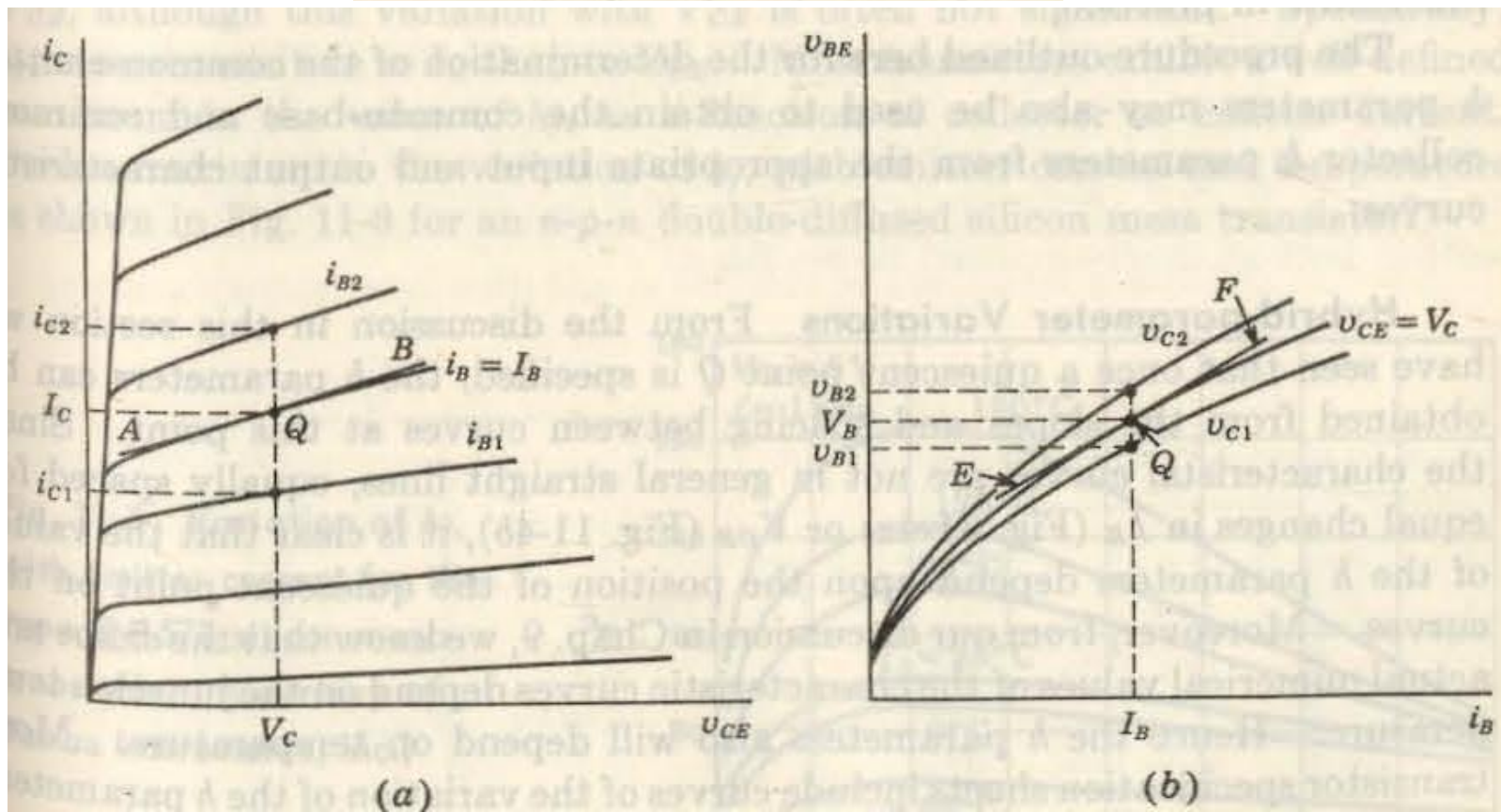
$$h_{ic} \quad | \quad h_{ie}$$

$$h_{ic} = h_{ie} \quad h_{rc} = 1 - h_{re} \quad h_{fc} = -(h_{fe} + 1) \quad h_{oc} = h_{oe}$$

DETERMINATION OF THE h PARAMETERS FROM THE CHARACTERISTICS²

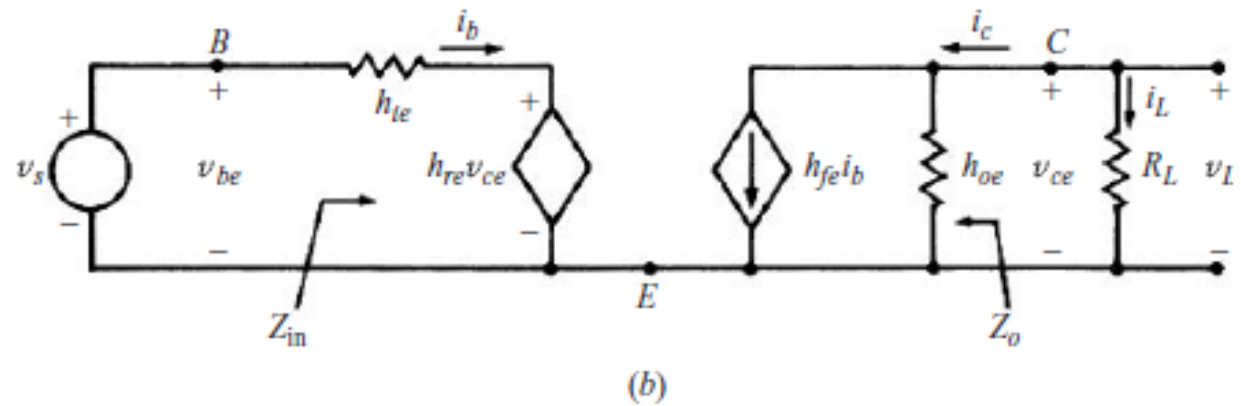
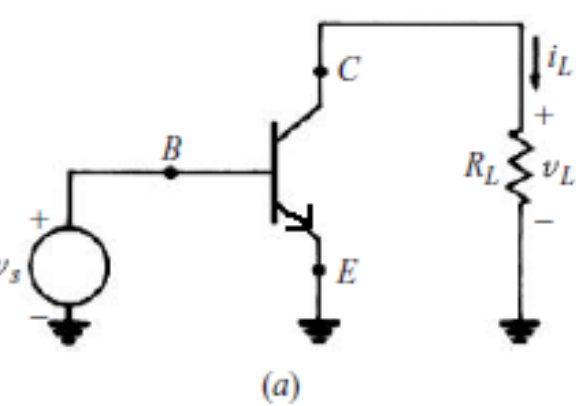
$$h_{fe} = \frac{\partial i_C}{\partial i_B} \approx \left. \frac{\Delta i_C}{\Delta i_B} \right|_{V_C} = \frac{i_{C2} - i_{C1}}{i_{B2} - i_{B1}}$$

$$h_{re} = \frac{\partial v_B}{\partial v_C} \approx \left. \frac{\Delta v_B}{\Delta v_C} \right|_{I_B} = \frac{v_{B2} - v_{B1}}{v_{C2} - v_{C1}}$$



$$h_{oe} = \frac{\partial i_C}{\partial v_C} \approx \frac{\Delta i_C}{\Delta v_C} \Big|_{I_B}$$

$$h_{ie} = \frac{\partial v_B}{\partial i_B} \approx \frac{\Delta v_B}{\Delta i_B} \Big|_{V_C}$$



$h_{ie} = 1 \text{ k}\Omega$, $h_{re} = 10^{-4}$, $h_{fe} = 100$, $h_{oe} = 12 \mu\text{S}$, and $R_L = 2 \text{ k}\Omega$.

$$A_i = \frac{i_L}{i_b} = -\frac{h_{fe}}{1 + h_{oe}R_L}$$

$$-\frac{100}{1 + (12 \times 10^{-6})(2 \times 10^3)} = -97.7$$

$$v_s = v_{be} = h_{ie}i_b + h_{re}v_{ce}$$

$$v_{ce} = -h_{fe}i_b \left(\frac{1}{h_{oe}} \parallel R_L \right) = \frac{-h_{fe}R_L i_b}{1 + h_{oe}R_L}$$

$$A_v = -\frac{h_{fe}R_L}{h_{ie} + R_L(h_{ie}h_{oe} - h_{fe}h_{re})}$$

$$= -\frac{(100)(2 \times 10^3)}{1 \times 10^3 + (2 \times 10^3)[(1 \times 10^3)(12 \times 10^{-6}) - (100)(1 \times 10^{-4})]} = -199.2$$

$$A_v \approx -h_{fe}R_L/h_{ie}$$

$$Z_{in} = \frac{v_s}{i_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L} = 1 \times 10^3 - \frac{(1 \times 10^{-4})(100)(2 \times 10^3)}{1 + (12 \times 10^{-6})(2 \times 10^3)} = 980.5 \Omega$$

$$Z_i \equiv \frac{V_1}{I_1}$$

$$V_1 = h_i I_1 + h_r V_2$$

$$Z_i = \frac{h_i I_1 + h_r V_2}{I_1} = h_i + h_r \frac{V_2}{I_1}$$

$$V_2 = -I_2 Z_L = A_I I_1 Z_L$$

$$Z_i = h_i + h_r A_I Z_L = h_i - \frac{h_f h_r}{Y_L + h_o}$$

$$Y_o \equiv \frac{I_2}{V_2} \quad \text{with } V_s = 0$$

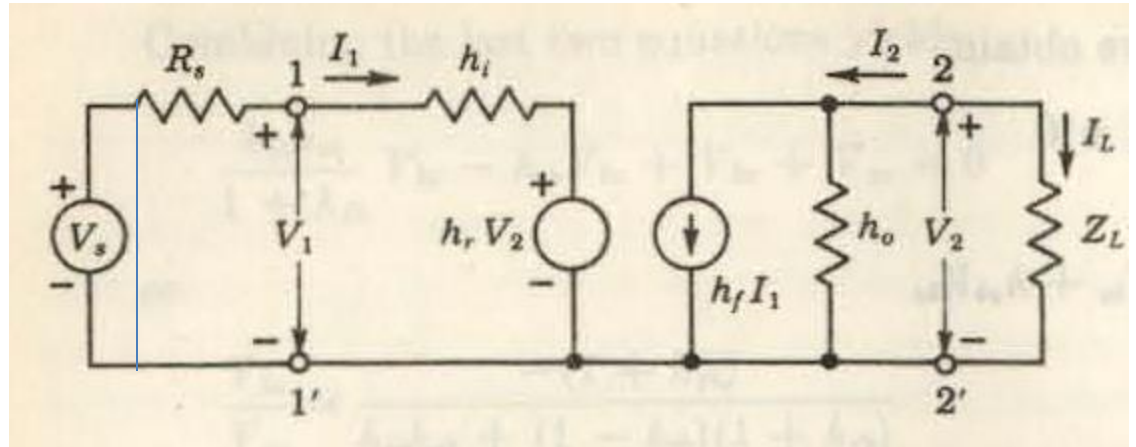
$$I_2 = h_f I_1 + h_o V_2$$

$$Y_o = h_f \frac{I_1}{V_2} + h_o$$

$$R_s I_1 + h_i I_1 + h_r V_2 = 0$$

$$\frac{I_1}{V_2} = - \frac{h_r}{h_i + R_s}$$

$$Y_o = h_o - \frac{h_f h_r}{h_i + R_s}$$



$$Z_o = 500 \text{ k}\Omega$$

1. Large current gain
2. Large voltage gain
3. Large power gain ($A_i A_v$)
4. Current and voltage phase shifts of 180°
5. Moderate input impedance
6. Moderate output impedance

In the CB amplifier of Fig. let $h_{ib} = 30 \Omega$, $h_{rb} = 4 \times 10^{-6}$, $h_{fb} = -0.99$, $h_{ob} = 8 \times 10^{-7} \text{ S}$, and $R_L = 20 \text{ k}\Omega$. (These are typical CB amplifier values.) Find expressions for the (a) current-gain ratio A_i , (b) voltage-gain ratio A_v , (c) input impedance Z_{in} , and (d) output impedance Z_o . (e) Evaluate this typical CE amplifier.

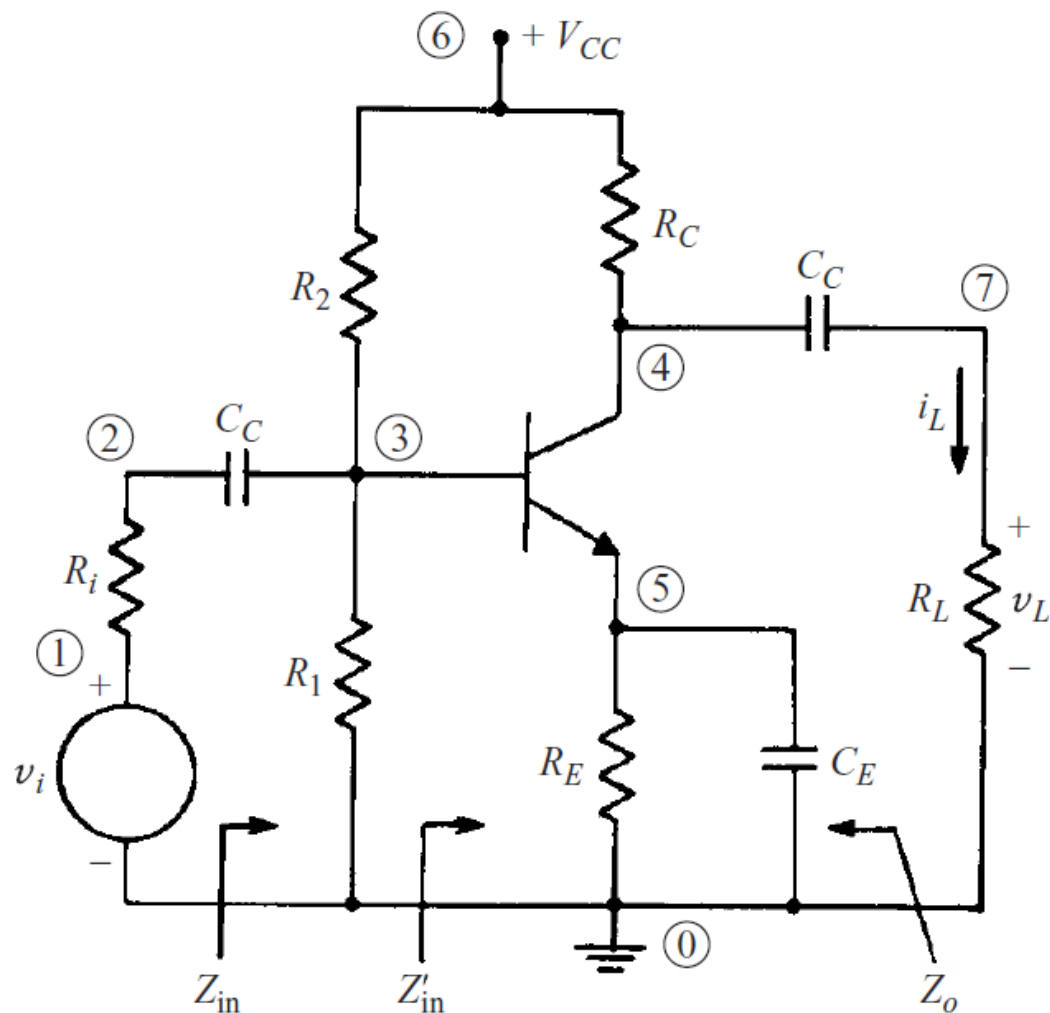
1. Current gain of less than 1
2. High voltage gain
3. Power gain approximately equal to voltage gain
4. No phase shift for current or voltage
5. Small input impedance
6. Large output impedance

In the CC amplifier of Fig. let $h_{ic} = 1 \text{ k}\Omega$, $h_{rc} = 1$, $h_{fc} = -101$, $h_{oc} = 12 \mu\text{S}$, and $R_L = 2 \text{ k}\Omega$.

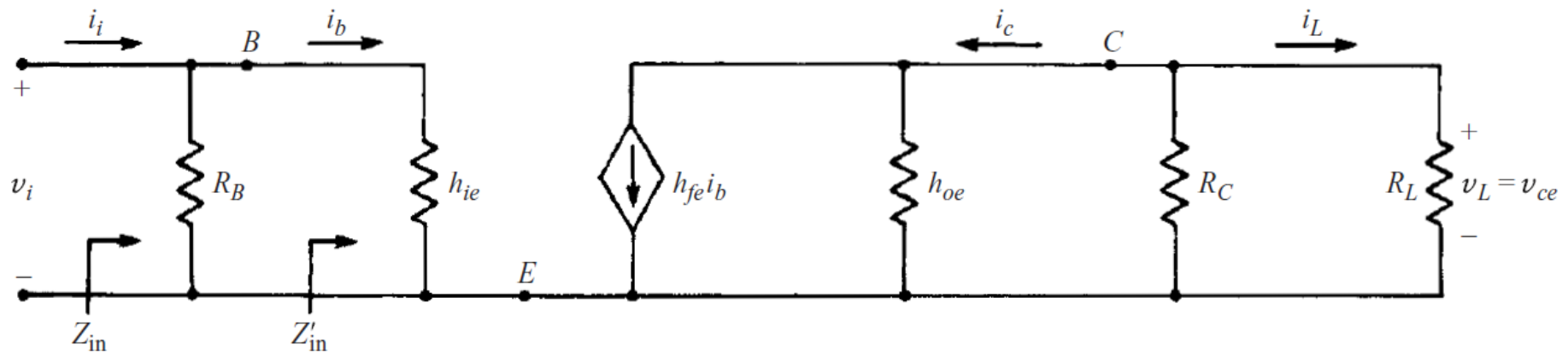
Drawing direct analogies with the CE amplifier of Example 6.2, find expressions for the (a) current-gain ratio A_i , (b) voltage-gain ratio A_v , (c) input impedance Z_{in} , and (d) output impedance Z_o . (e) Evaluate this typical CC amplifier.

1. High current gain
2. Voltage gain of approximately unity
3. Power gain approximately equal to current gain
4. No current or voltage phase shift
5. Large input impedance
6. Small output impedance

Use a small-signal h -parameter equivalent circuit to analyze the amplifier of Fig. 3-10(a), given $R_C = R_L = 800 \Omega$, $R_i = 0$, $R_1 = 1.2 \text{ k}\Omega$, $R_2 = 2.7 \text{ k}\Omega$, $h_{re} \approx 0$, $h_{oe} = 100 \mu\text{S}$, $h_{fe} = 90$, and $r_{ie} = 200 \Omega$. Calculate (a) the voltage gain A_v and (b) the current gain A_i .



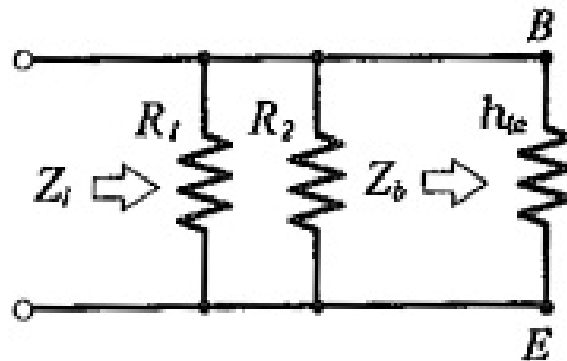
(a)



Small signal equivalent circuit

$$R_B = R_1 R_2 / (R_1 + R_2) = 831 \Omega.$$

$$R_L' = R_C \parallel R_L = 400 \Omega$$



$$A_i = \frac{i_L}{i_b} = -\frac{h_f}{1 + h_o R_L}$$

$$A_v = -\frac{h_f R_L}{h_i + R_L(h_i h_o - h_f h_r)}$$

$$Z_i = h_i - \frac{h_f h_r}{Y_L + h_o}$$

$$Y_o = h_o - \frac{h_f h_r}{R_S + h_i}$$

$$A_v = -\frac{\frac{h_f}{h_i h_o} h_o R_L}{1 + R_L h_o - \frac{h_f h_r R_L}{h_i}} = \frac{A_{vo} h_o R_L}{1 + R_L h_o + A_{vo} h_o h_r R_L}$$

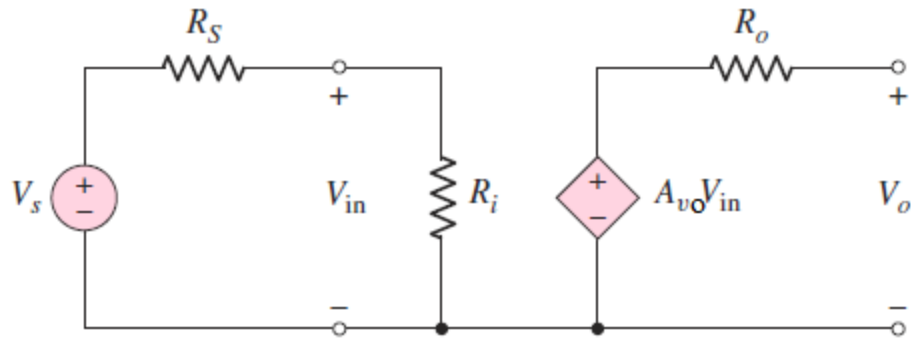
If there is a source resistance R_s ,

$$v_{in} = \frac{Z_i}{Z_i + R_S} v_S$$

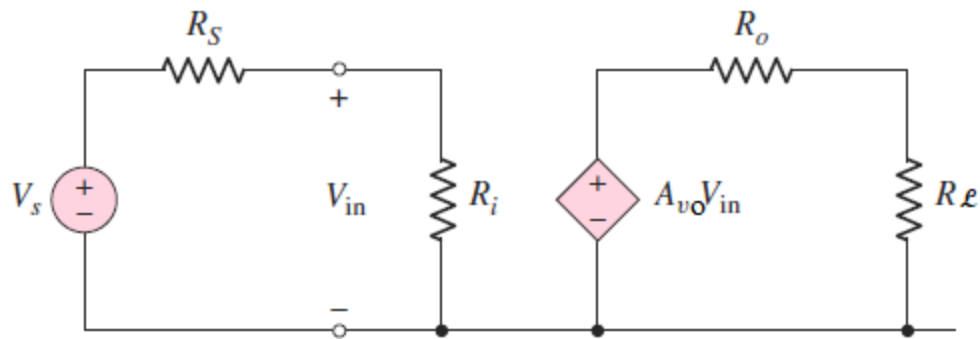
$$A_{vS} = \frac{Z_i}{Z_i + R_S} A_v$$

If there is a parallel resistance R_B

$$i_b = \frac{R_B}{R_B + h_{ie}} i_i \quad A_{iS} = \frac{i_L}{i_i} = \frac{R_B}{R_B + h_{ie}} A_i$$



Two-port equivalent circuit for the amplifier

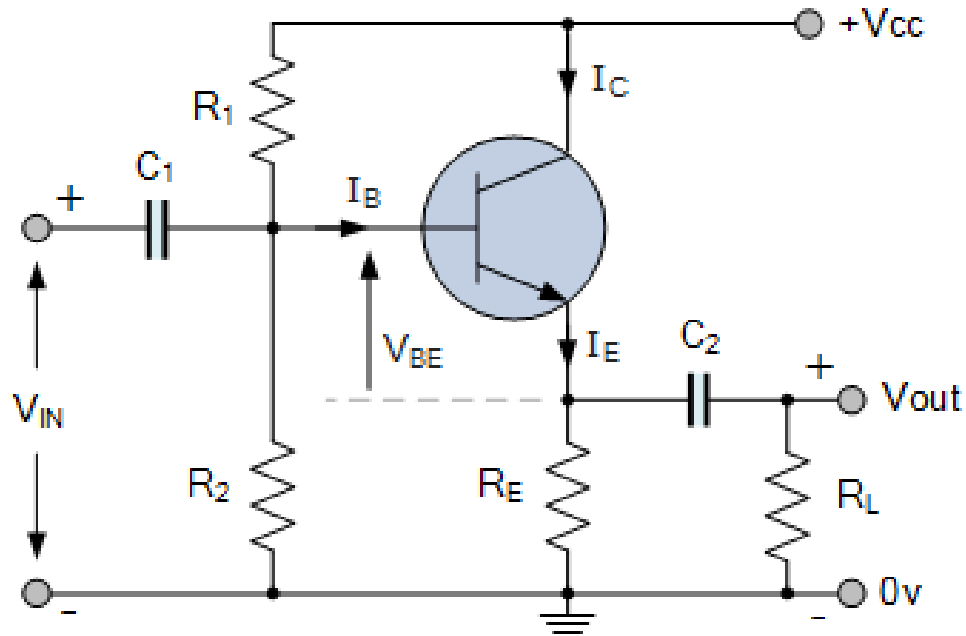


Two-port equivalent circuit for the amplifier

$$A_v = A_{vo} R_L / (1/h_o + R_L)$$

$$= A_{vo} h_o R_L / (1 + h_o R_L)$$

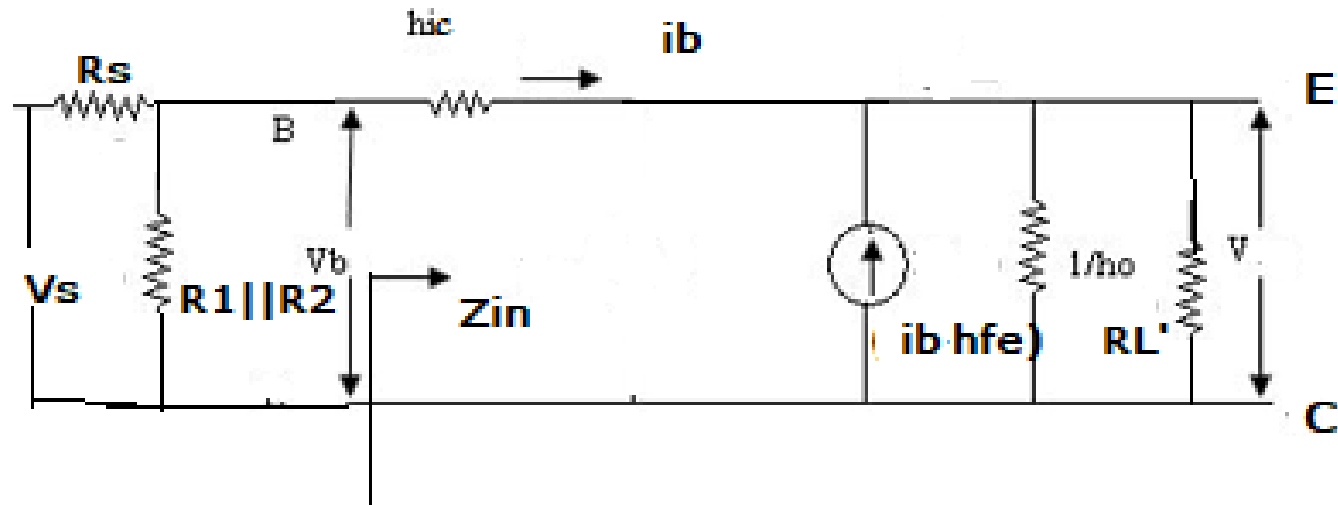
$$A_i = \frac{i_L}{i_b} = -\frac{h_f}{1 + h_o R_L} = (1+h_{fe})/(1+h_{oe}R_L)$$



$$R_L' = R_E \parallel R_L$$

$$Z_{ic} = h_{ie} + \frac{(1 + h_{fe})}{Y_L + h_{oe}}$$

$$h_{ie} + (1 + h_{fe})(R_L' \parallel 1/h_{oe})$$

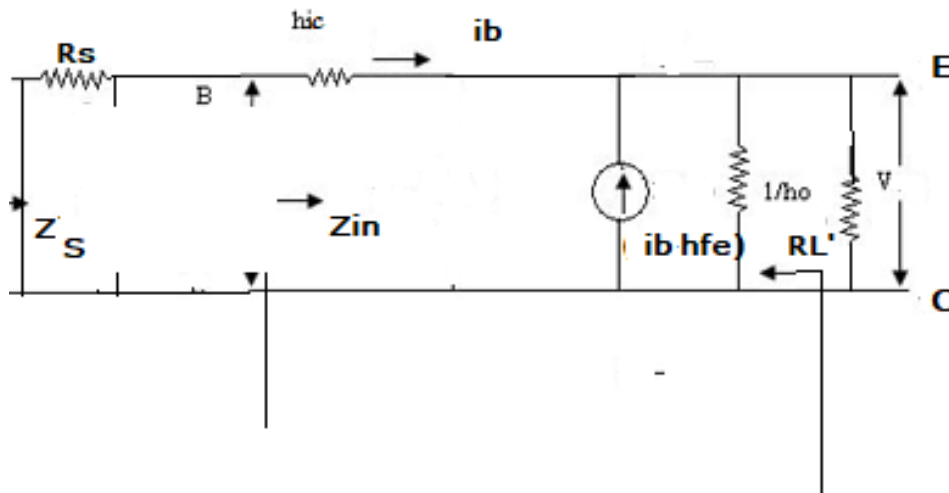


$$Z_{ib} = h_{ie} / (1 + h_{fe})$$

$$v_b = h_{ie} i_b + (R_L' \parallel 1/h_{oe})(1 + h_{fe}) i_b$$

$$Z_{in} = v_b / i_b = h_{ie} + (R_L' \parallel 1/h_{oe})(1 + h_{fe})$$

$$v_{in} = v_b = (Z_{in}' / (Z_{in}' + R_S)) v_s$$



$$(R_S + h_{ie}) i_b + v_o = 0$$

$$i_b / v_o = -1 / (R_S + h_{ie})$$

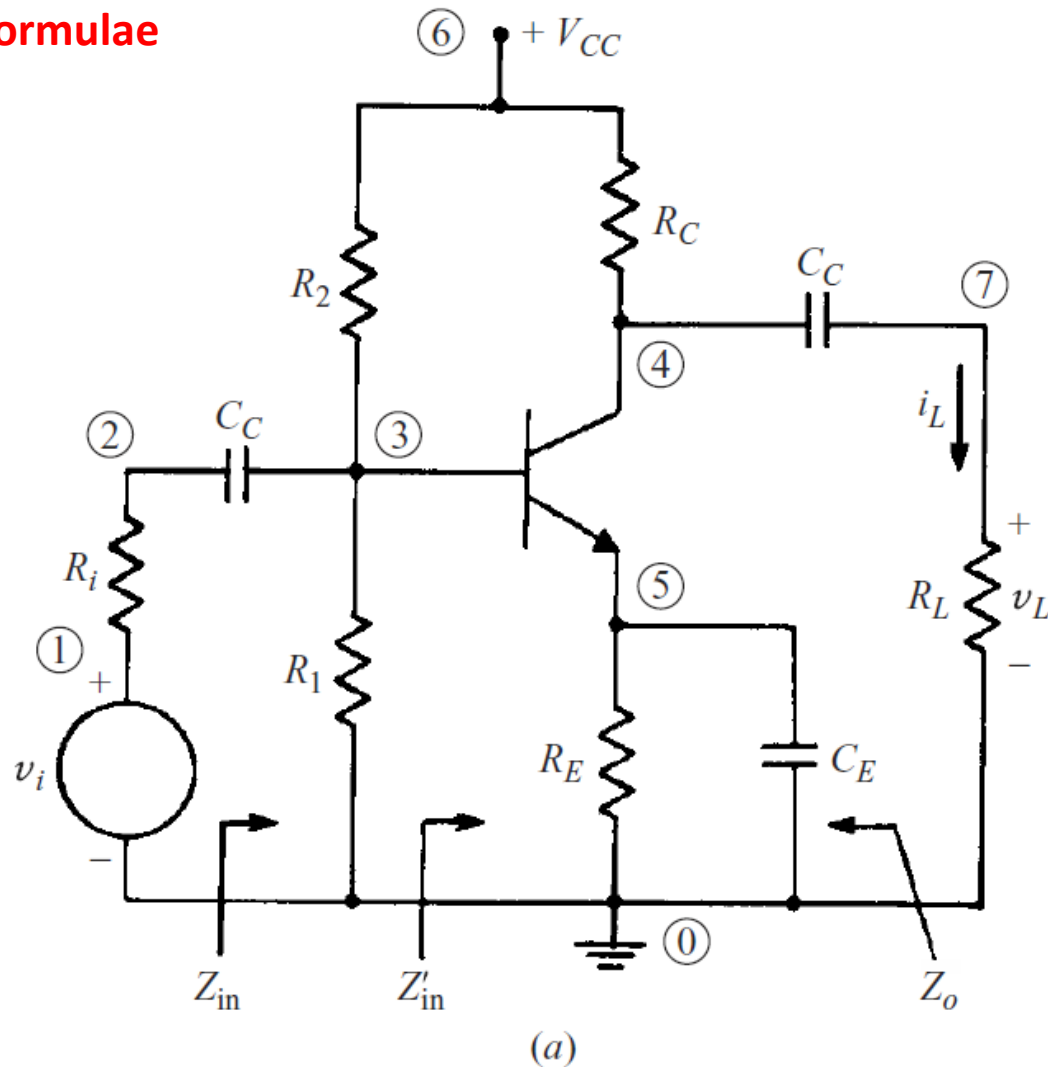
$$i_o = (h_{fe} + 1) i_b + h_{oe} v_o$$

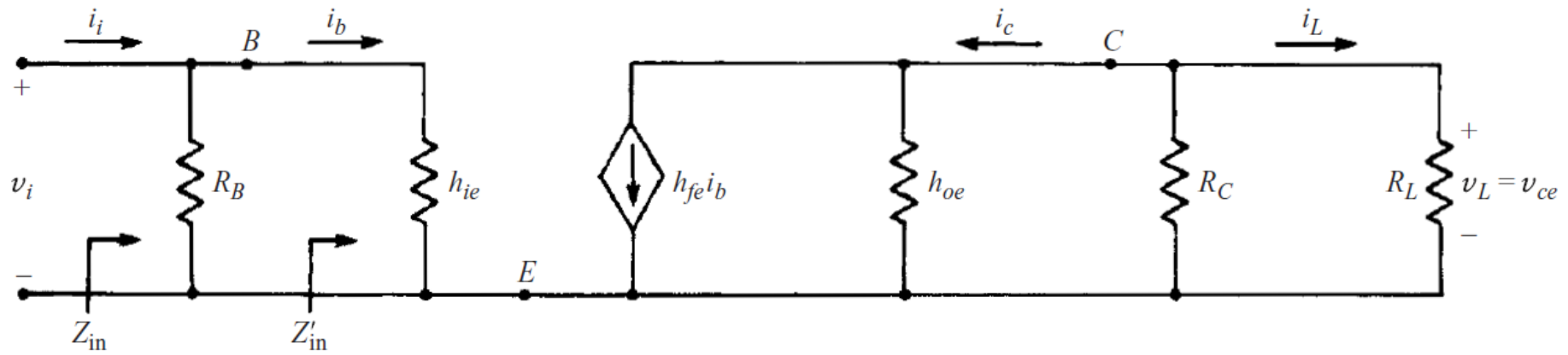
$$i_o / v_o = (h_{fe} + 1) i_b / v_o + h_{oe}$$

$$Y_o = -(h_{fe} + 1) / (R_S + h_{ie}) + h_{oe}$$

Use a small-signal h -parameter equivalent circuit to analyze the amplifier of Fig. 3-10(a), given $R_C = R_L = 800 \Omega$, $R_i = 0$, $R_1 = 1.2 \text{ k}\Omega$, $R_2 = 2.7 \text{ k}\Omega$, $h_{re} \approx 0$, $h_{oe} = 100 \mu\text{S}$, $h_{fe} = 90$, and $r_{ie} = 200 \Omega$. Calculate (a) the voltage gain A_v and (b) the current gain A_i .

Do not use formulae





Small signal equivalent circuit

$$R_B = R_1 R_2 / (R_1 + R_2) = 831 \Omega.$$

$$R_L' = R_C \parallel R_L = 400 \Omega$$

$$A_{v_o} = \frac{-h_{fe}}{h_{ie} h_{oe}} = -\frac{90}{200 \times 100 \times 10^{-6}} = -4.5 \times 10^3$$

$$A_v = \frac{A_{v_o} h_o R_L}{1 + R_L h_o + A_{v_o} h_o h_r R_L} = -4.5 \times 10^3 \times \frac{100 \times 10^{-6} \times 400}{1 + 100 \times 10^{-6} \times 400}$$

$$= -45 \times \frac{4}{1+0.04} = -173.08$$

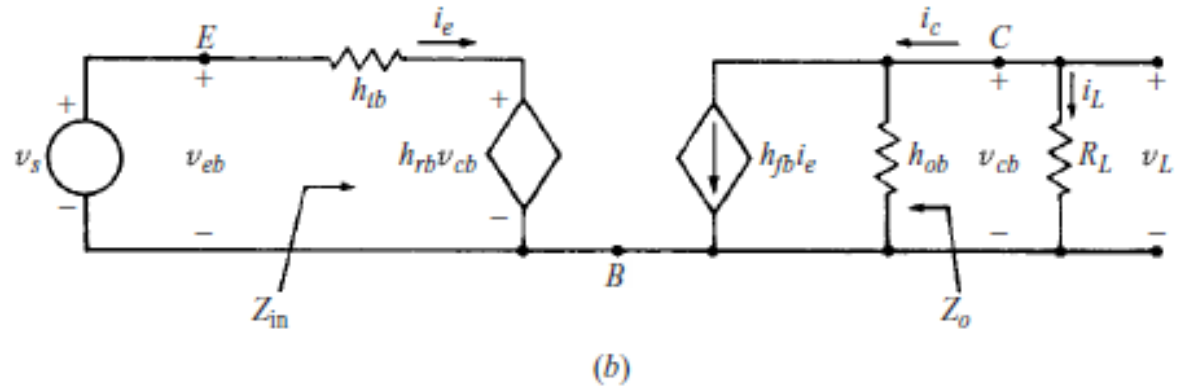
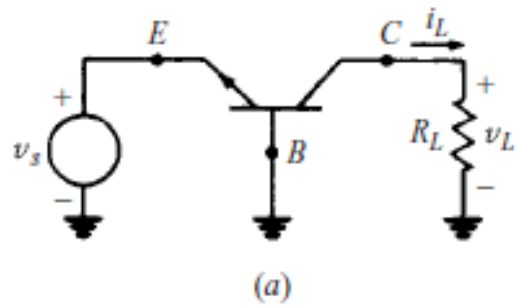
$$i_b = \frac{R_B}{R_B + h_{ie}} i_i$$

$$A_i = \frac{i_L}{i_b} = -\frac{h_f}{1 + h_o R_L}$$

$$A_i = \frac{i_L}{i_b} = \frac{-h_{fe}}{1 + R_L h_{oe}} = -\frac{90}{1 + 800 \times 100 \times 10^{-6}} = -43.27$$

$$A_{is} = \frac{i_L}{i_i} = \frac{R_B}{R_B + h_{ie}} A_i = -\frac{831}{831 + 200} 43.27 = -34.87$$

Effect of coupling capacitance and bypass capacitance



$$h_{ib} = h_{ie} / (1 + h_{fe})$$

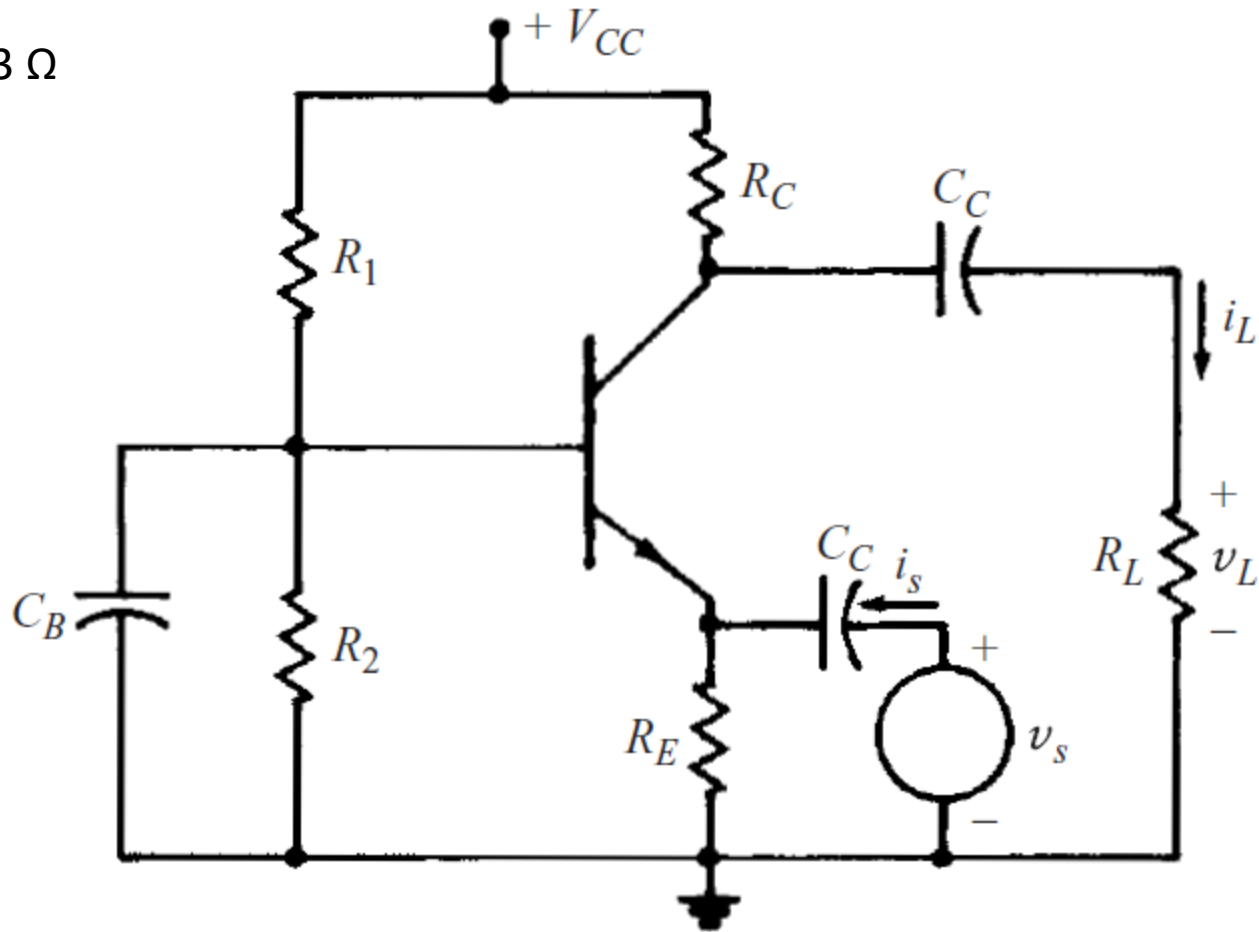
$$h_{rb} = -h_{re}$$

$$h_{fb} = -h_{fe} / (1 + h_{fe})$$

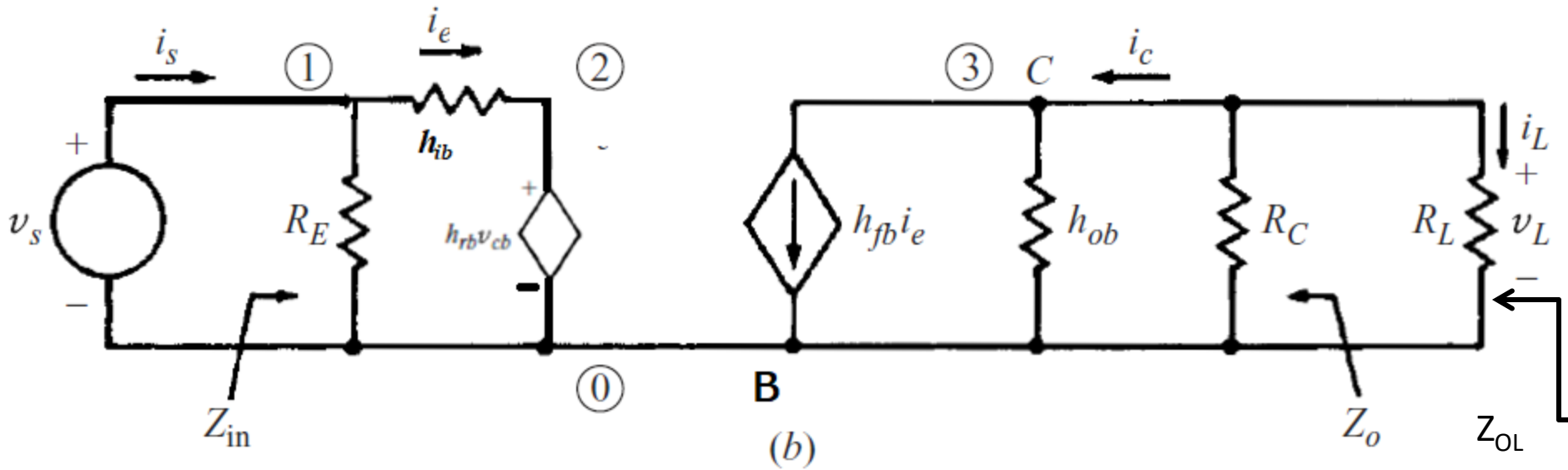
$$h_{ob} = h_{oe} / (1 + h_{fe})$$

In the CB amplifier of Fig (a), let $R_1 = R_2 = 50 \text{ k}\Omega$, $R_C = 2.2 \text{ k}\Omega$, $R_E = 3.3 \text{ k}\Omega$, $R_L = 1.1 \text{ k}\Omega$, $C_C = C_B \rightarrow \infty$, $h_{rb} \approx 0$, $h_{ib} = 25 \text{ }\Omega$, $h_{ob} = 10^{-6} \text{ S}$, and $h_{fb} = -0.99$. Find and evaluate expressions for (a) the voltage-gain ratio $A_v = v_L/v_s$ and (b) the current-gain ratio $A_i = i_L/i_s$.

$$R_C \parallel R_L = 733 \text{ }\Omega$$



(a)



$$Z_{OL} = 1/h_{ob} \parallel R_C \parallel R_L = 732.8 \Omega$$

$$v_L = -h_{fb} i_e Z_{OL}$$

$$v_s = v_i = h_{ib} i_e$$

$$A_v = -h_{fb} Z_{OL} / h_{ib}$$

$$= 0.99 \times 732.8 / 25 = 29.02$$

$$i_L = v_L / R_L = -h_{fb} i_e Z_{OL} / R_L$$

$$A_i = i_L / i_e = -h_{fb} Z_{OL} / R_L$$

$$A_i = A_v \times h_{ib} / R_L = 29.02 \times 25 / 1100 = 0.6595$$

$$A_{is} = 3.3 / (3.3 + 0.025) \times 0.6595 = 0.6545$$

$$Z_{in} = h_{ib} \parallel R_E$$

$$= 24.8 \Omega$$

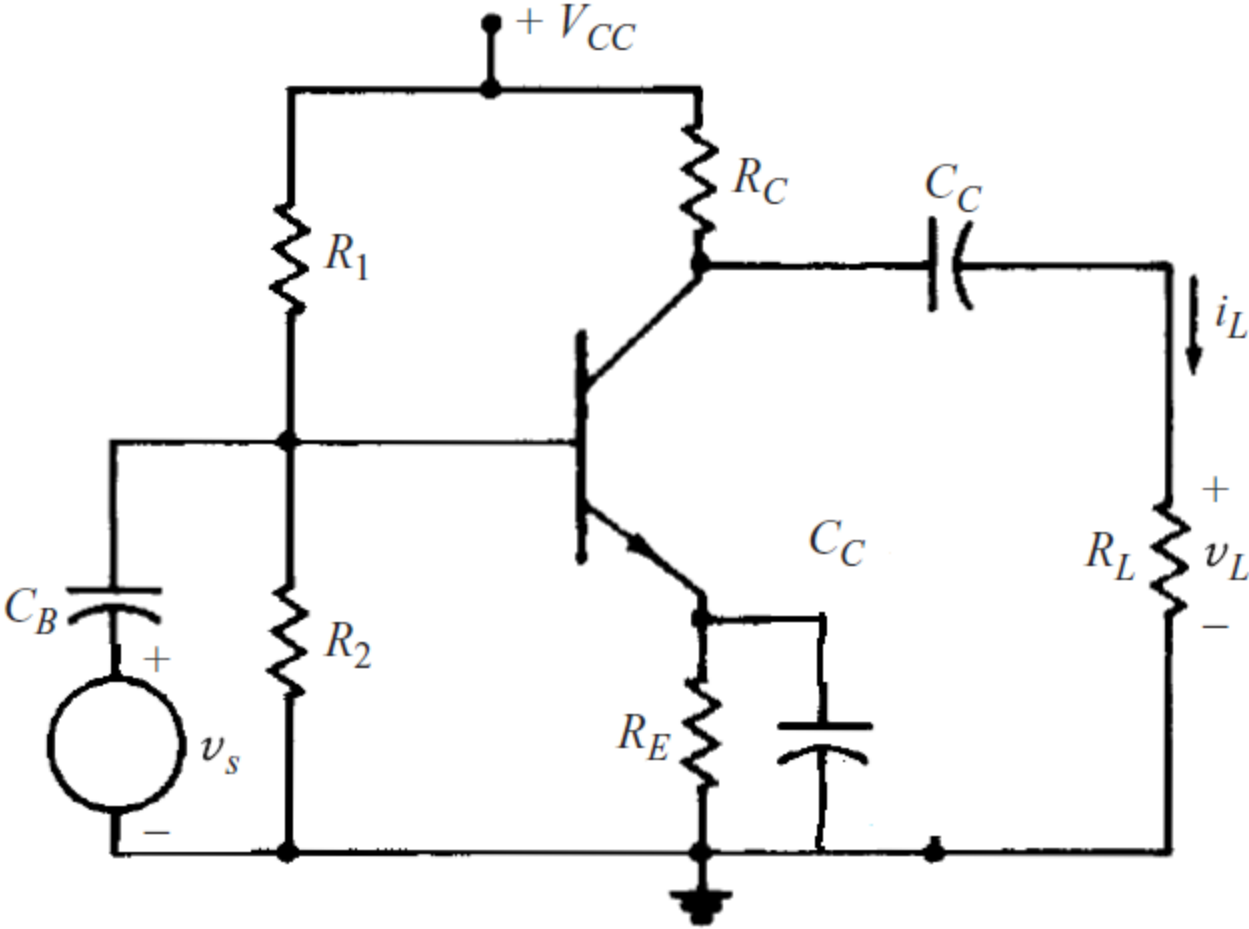
$$Z_o = 1/h_{ob} \parallel R_C$$

$$= 2.195 \text{ k}\Omega$$

$$i_e = \frac{R_E}{R_E + h_{ib}} i_s$$

$$A_{is} = \frac{i_L}{i_s} = \frac{R_E}{R_E + h_{ib}} A_i$$

Same Circuit in CE mode



(a)

$$hfb = -hfe / (1 + hfe) \rightarrow hfe = -hfb / (1 + hfb) = -0.99 / (1 - 0.99) = 99$$

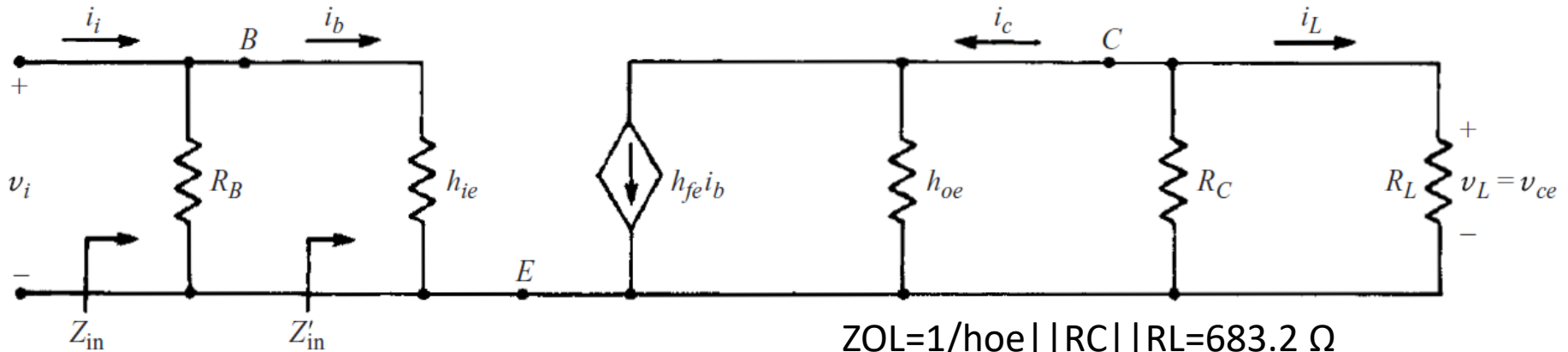
$$hib = hie / (1 + hfe) \rightarrow hie = 25 * 100 = 2500 \Omega$$

$$hrb = -hre \rightarrow hre = 0$$

$$hob = hoe / (1 + hfe) \rightarrow hoe = 10^{-4} \text{ S}$$

$$R_B = R1 || R2$$

$$= 25 \text{ k}\Omega$$



$$Z_{in} = h_{ie} || R_B$$

$$= 2.252 \text{ k}\Omega$$

$$Z_o = 1/h_{oe} || R_c$$

$$= 1.8 \text{ k}\Omega$$

$$Z_{OL} = 1/h_{oe} || R_C || R_L = 683.2 \Omega$$

$$V_L = -h_{fe} i_b Z_{OL}$$

$$V_s = v_i = h_{ie} i_b$$

$$A_v = -h_{fe} Z_{OL} / h_{ie}$$

$$= -99 * 683.2 / 2500 = -27.05$$

$$i_L = v_L / R_L = -h_{fe} i_b Z_{OL} / R_L$$

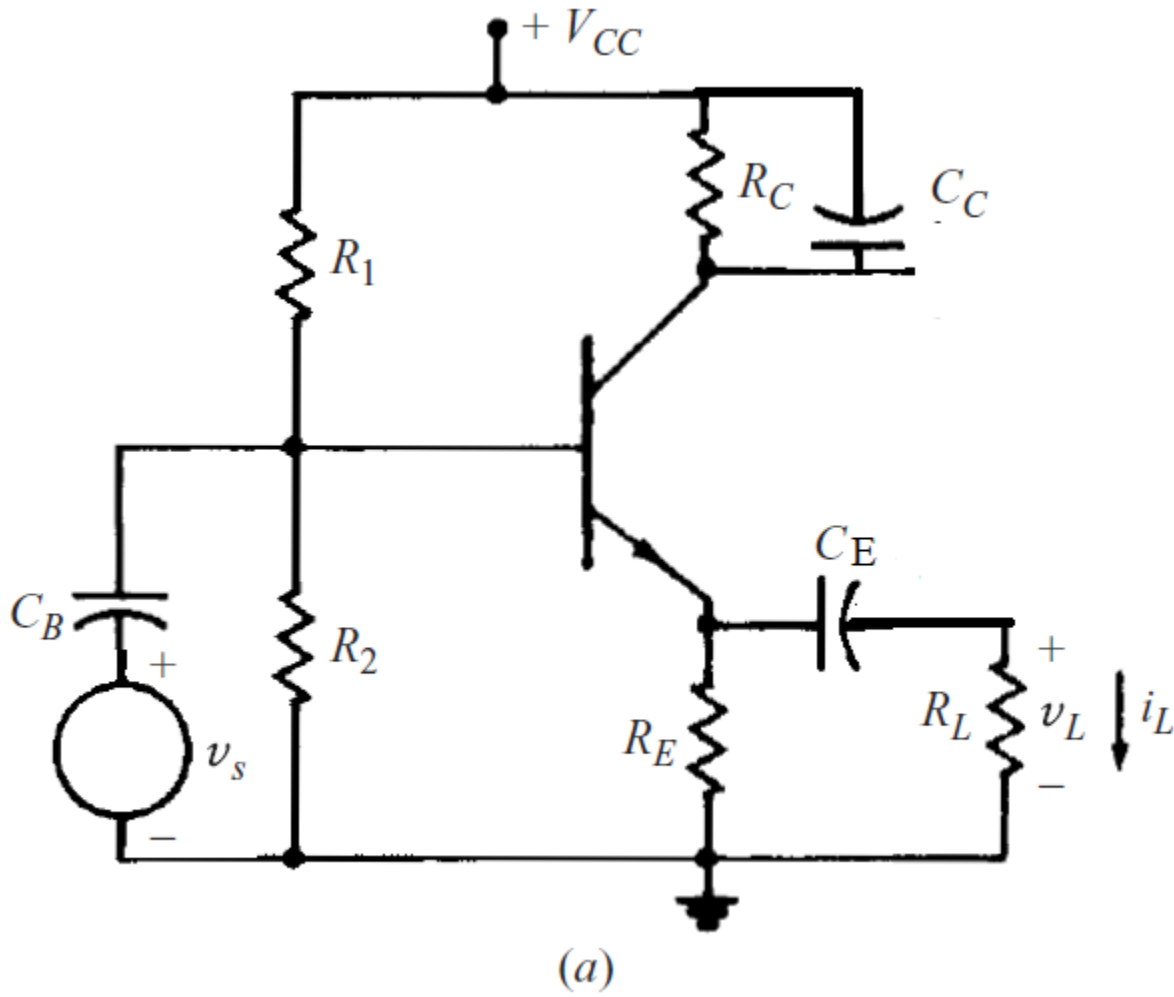
$$A_i = i_L / i_b = -h_{fe} Z_{OL} / R_L$$

$$A_i = A_v * h_{ie} / R_L = -27.05 * 2500 / 1100 = -61.49$$

$$A_{is} = -25 / (25 + 2.475) * 61.49 = -55.95$$

$$A_{is} = \frac{i_L}{i_s} = \frac{R_B}{R_B + h_{ie}} A_i$$

Same Circuit in CC mode

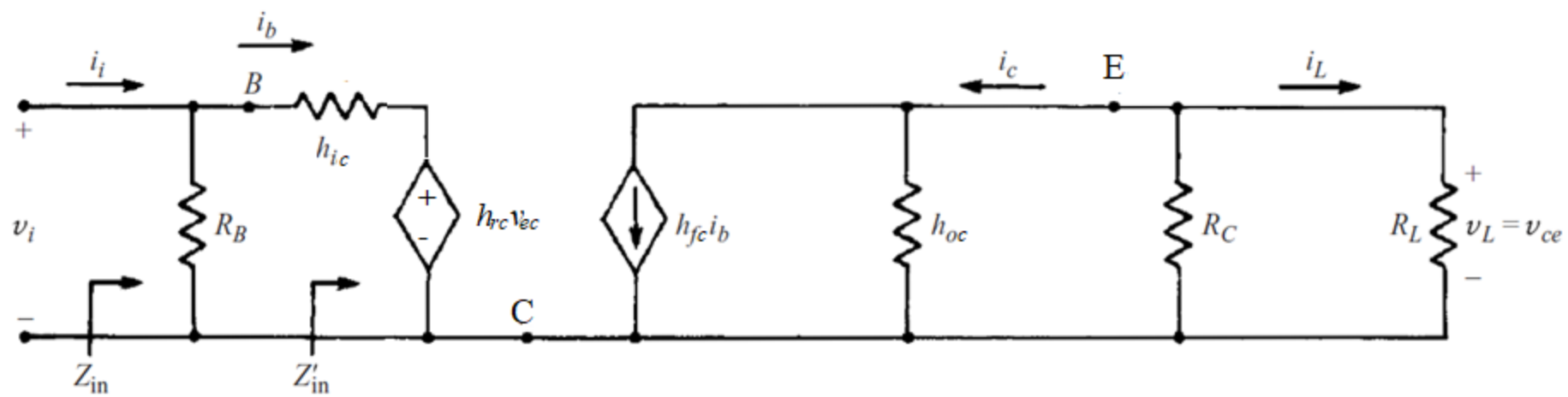


$$h_{ic} = h_{ie} = 2500 \Omega$$

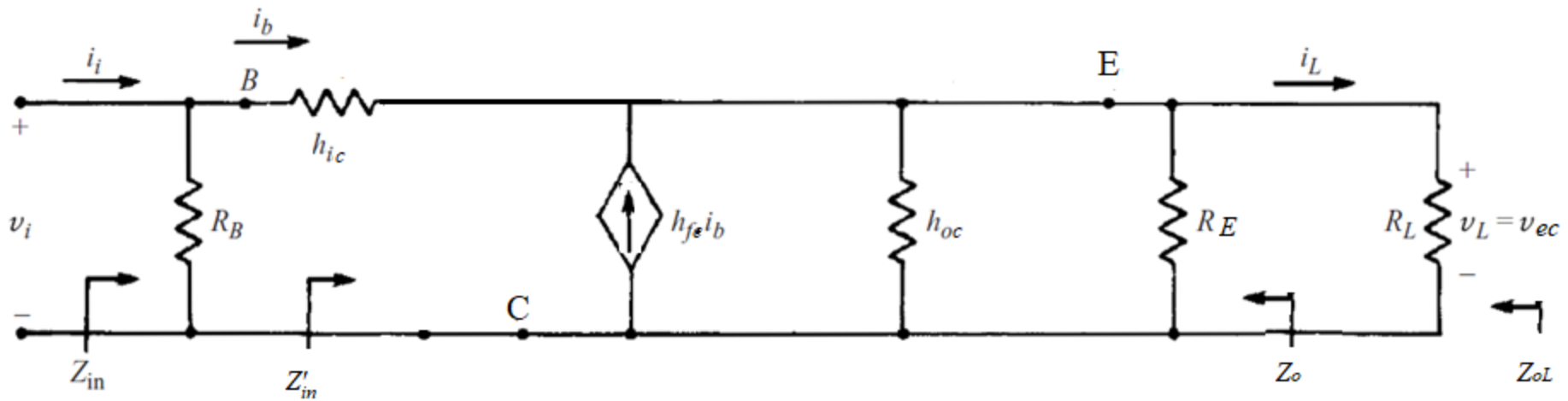
$$h_{rc} = 1 - h_{re} = 1$$

$$h_{fc} = -(1 + h_{fe}) = -100$$

$$h_{oc} = h_{oe} = 10^4 \Omega = 10 \text{ K}\Omega$$



|||



$$Z'_{in} = h_{ic} + \frac{(1 + h_{fe})}{Y_L + h_{oe}} = 2.5 + 100 \times (R_E \parallel 1/h_{oe})$$

$$= 2.5 + 100 \times 3.3 \parallel 10$$

$$= 2.5 + 100 \times 2.481$$

$$= 250.575 \text{ k}\Omega$$

Without considering \$R_L\$

$$Z_{in} = R_B \parallel Z'_{in} = 25 \parallel 250.575 = 22.73 \text{ k}\Omega$$

Considering \$R_L\$

$$Z'_{inL} = 2.5 + 100 \times (R_E \parallel R_L \parallel 1/h_{oe})$$

$$= 2.5 + 100 \times (3.3 \parallel 1.1 \parallel 10)$$

$$= 2.5 + 100 \times 0.762$$

$$= 78.7 \text{ k}\Omega$$

$$Z_{inL} = R_B \parallel Z'_{inL} = 25 \parallel 78.7 = 18.97 \text{ k}\Omega$$

$$Z_o = R_E \parallel 1/h_{oc} \parallel [h_{ie}/(1+h_{fe})]$$

$$= 3.3 \parallel 10 \parallel 0.025 \text{ k}\Omega = 24 \Omega$$

$$\text{Let } Z = 1/h_{oc} \parallel R_E \parallel R_L = 762 \Omega$$

$$V_L = (1+h_{fe})i_b \times Z$$

$$V_s = v_i = Z'_{inL} i_b$$

$$A_v = (1+h_{fe}) \times Z / Z'_{inL}$$

$$= 100 \times 0.762 / 78.675 = 0.968$$

$$i_L = v_L / R_L = (1+h_{fe})i_b \times Z / R_L$$

$$A_i = i_L / i_b = (1+h_{fe}) \times Z / R_L$$

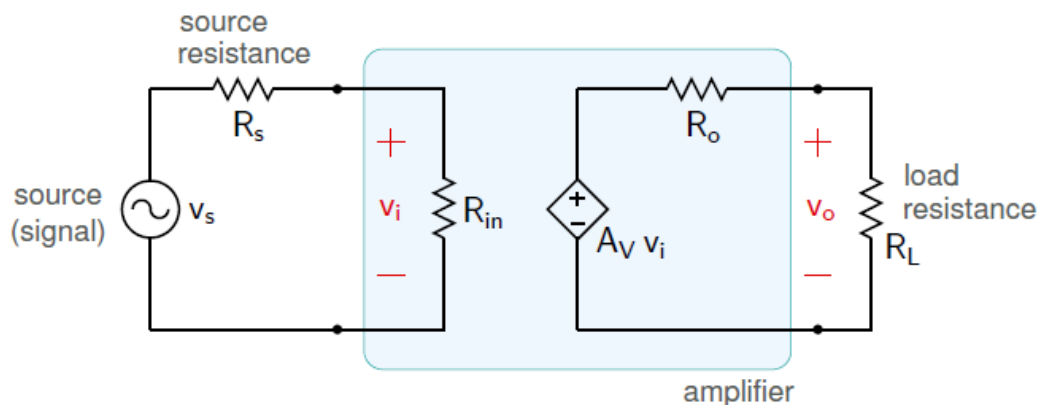
$$A_i = A_v \times Z'_{inL} / R_L = 0.968 \times 78.675 / 1.1 = 69.27$$

$$A_{is} = -25 / (25 + Z'_{inL}) \times 69.27 = 16.7$$

Consider $v_s = 20\sin(2000\pi t)$ mV

Draw input and output voltage and current waveforms with proper level

General representation of an amplifier



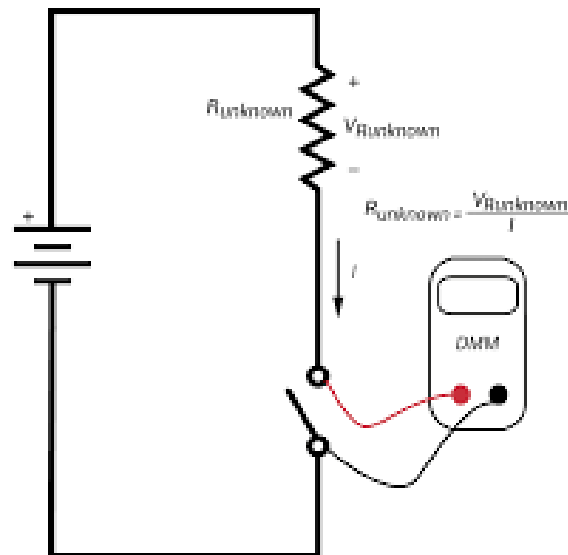
- * An amplifier is represented by a voltage gain, an input resistance R_{in} , and an output resistance R_o . For a voltage-to-voltage amplifier, a large R_{in} and a small R_o are desirable.
- * The above representation involves AC quantities *only*, i.e., it describes the AC equivalent circuit of the amplifier.
- * The DC bias of the circuit can affect parameter values in the AC equivalent circuit (A_V , R_{in} , R_o). For example, for the common-emitter amplifier, $A_V \propto g_m = I_C/V_T$, I_C being the DC (bias) value of the collector current.
- * Suppose we are given an amplifier as a “black box” and asked to find A_V , R_{in} , and R_o . What experiments would give us this information?

Measurement of Voltage gain

If $R_L \rightarrow \infty$, $i_l \rightarrow 0$, and $v_o \rightarrow A_V v_i$.

We can remove R_L (i.e., replace it with an open circuit), measure v_i and v_o , then use $A_V = v_o/v_i$.

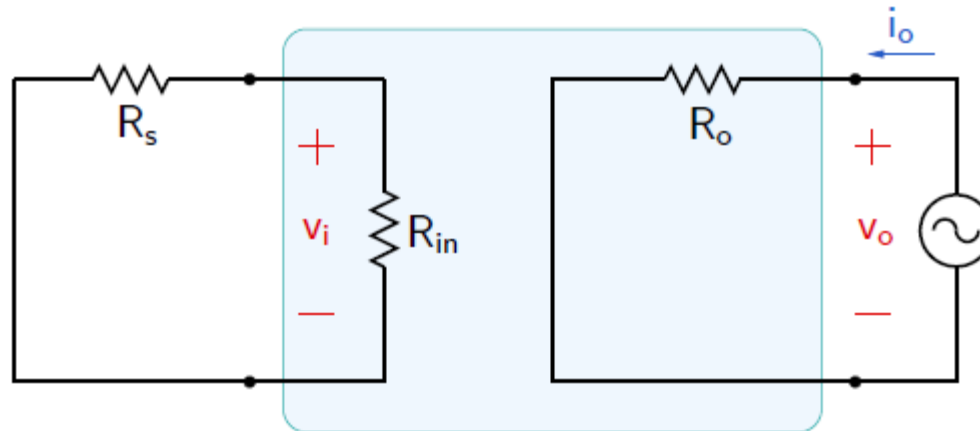
Measurement of v_i and i_i yields $R_{in} = v_i/i_i$.

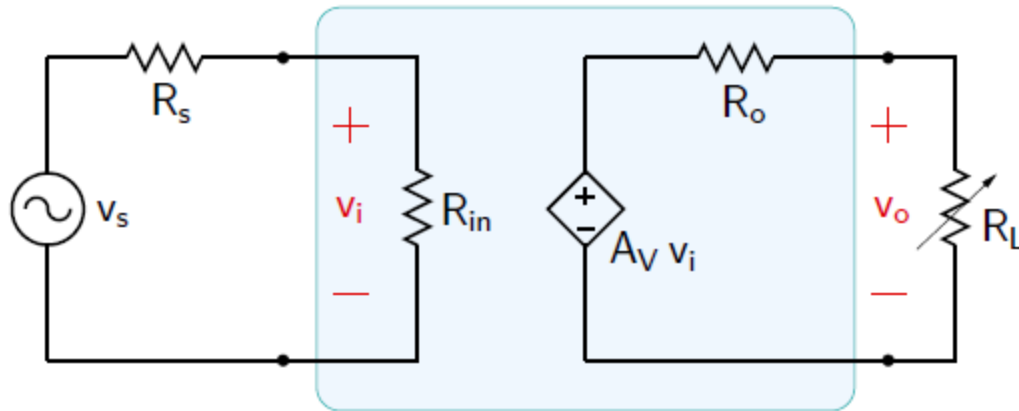


Method 1:

If $v_s \rightarrow 0$, $A_V v_i \rightarrow 0$.

Now, connect a test source v_o , and measure $i_o \rightarrow R_o = v_o/i_o$.





$$v_o = \frac{R_L}{R_L + R_o} A_V v_i.$$

$$\text{If } R_L \rightarrow \infty, v_{o1} = A_V v_i.$$

$$\text{If } R_L = R_o, v_{o2} = \frac{1}{2} A_V v_i = \frac{1}{2} v_{o1}.$$

Procedure:

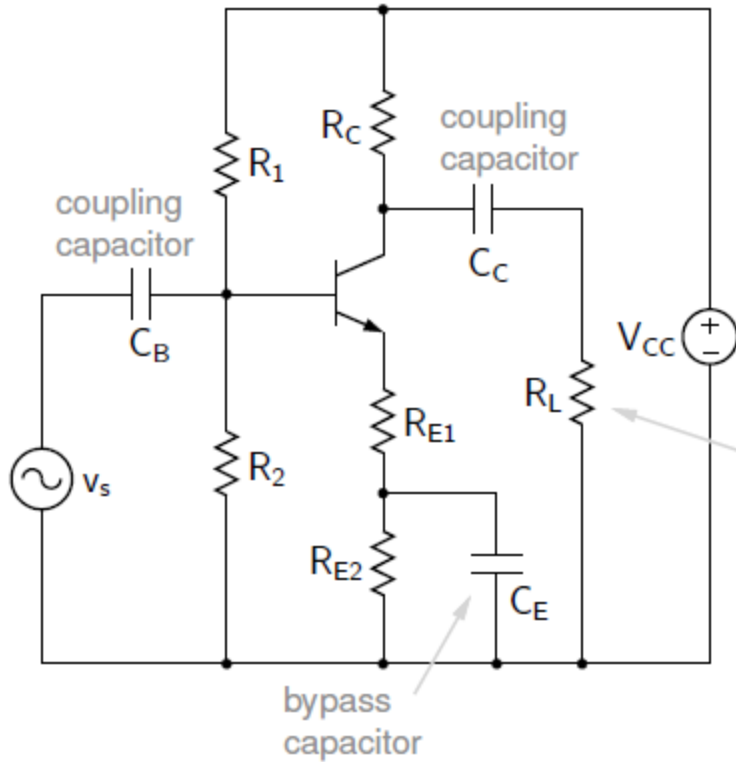
Measure v_{o1} with $R_L \rightarrow \infty$ (i.e., R_L removed).

Vary R_L and observe v_o .

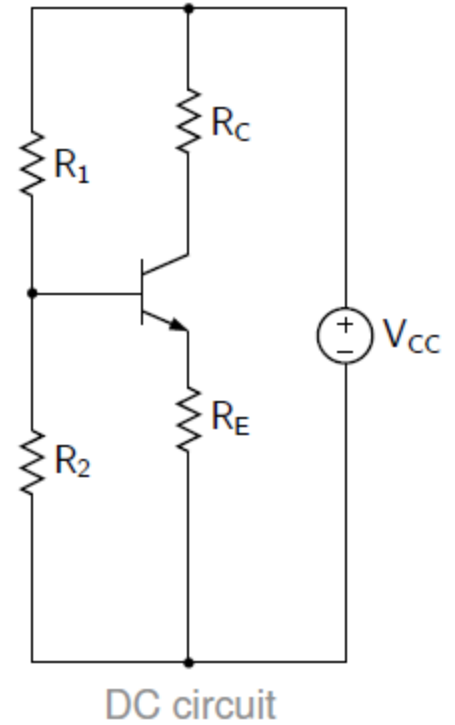
When v_o is equal to $v_{o1}/2$, measure R_L (after removing it).

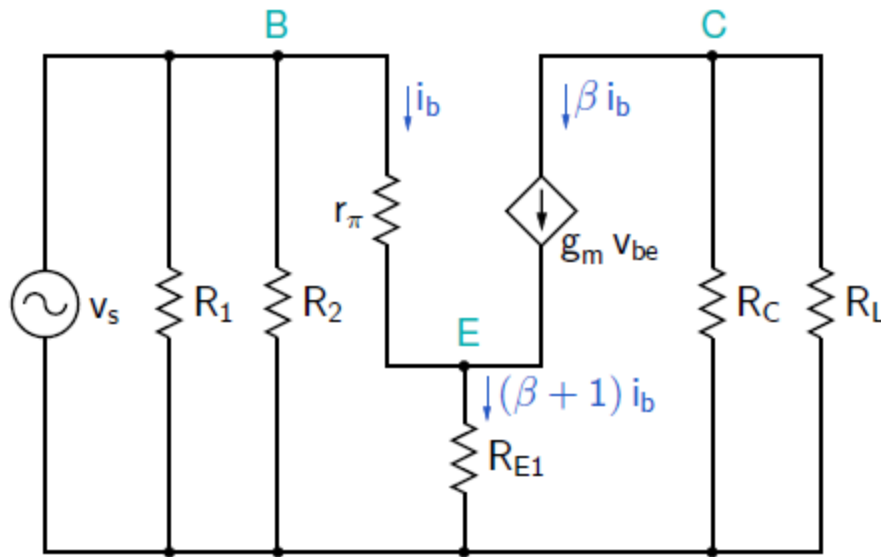
R_o is the same as the measured resistance.

Common-emitter amplifier with partial bypass



load resistor





AC circuit

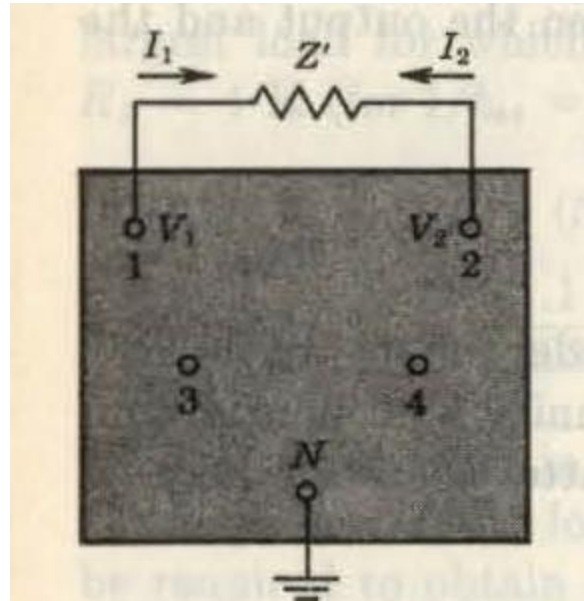
$$v_s = i_b r_\pi + (\beta + 1) i_b R_{E1} \rightarrow i_b = \frac{v_s}{r_\pi + (\beta + 1) R_{E1}}$$

$$v_o = -\beta i_b \times (R_C \parallel R_L) \rightarrow \frac{v_o}{v_s} = -\frac{\beta (R_C \parallel R_L)}{r_\pi + (\beta + 1) R_{E1}} = -R_C \parallel R_L / (r_\pi / \beta + R_{E1})$$

$$= -R_C \parallel R_L / (r_e + R_{E1})$$

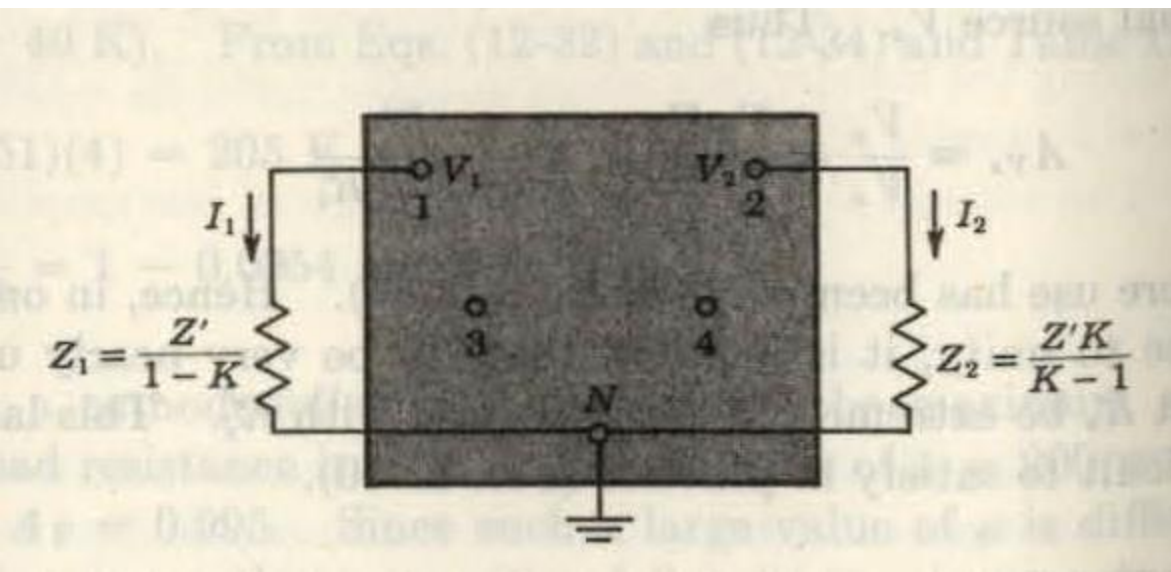
$$= -R_C / R_{E1}$$

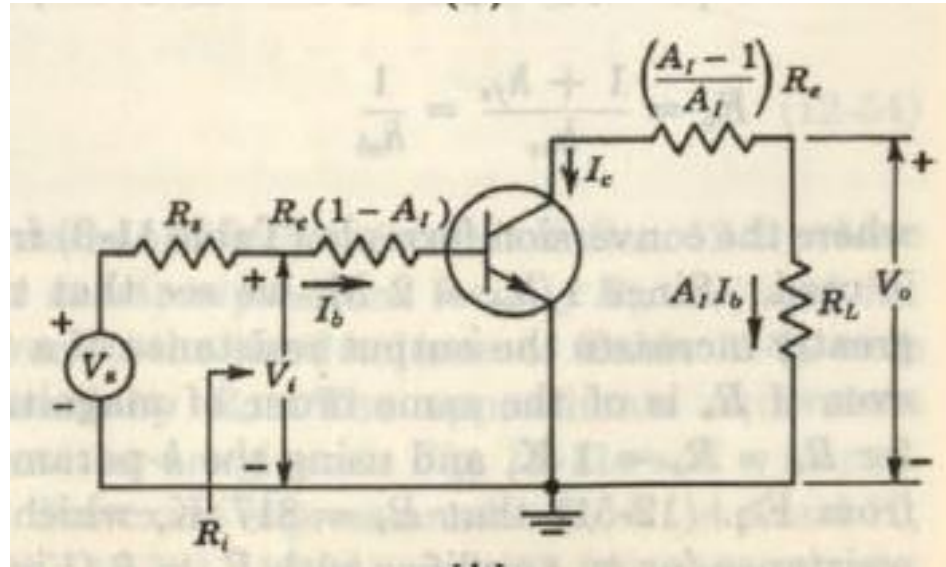
MILLER'S THEOREM



$$I_1 = \frac{V_1 - V_2}{Z'} = \frac{V_1(1 - K)}{Z'} = \frac{V_1}{Z'/(1 - K)} = \frac{V_1}{Z_1}$$

$$Z_2 \equiv \frac{Z'}{1 - 1/K} = \frac{Z'K}{K - 1}$$





Mayukhmali Das

12:34 PM

Yes mam

Mayukhmali Das

12:42 PM

Mam, many students are trying to join

Please allow them

Mayukhmali Das

2:41 PM

Ok mam

Mayukhmali Das

2:54 PM

98

Mam roll number 98

Sandip Dutta

2:55 PM

Sandip Dutta

Sagnik Das

2:55 PM

75

Snehasish Roy

2:55 PM

Snehasish Roy 001910701013

Aditya Bikram Mitra

2:55 PM

83 present

Souvik Barman

2:55 PM

Souvik Barman Roll no 76

Sandip Dutta

2:55 PM

Sandip Dutta - 17

Keshav Jasrotia

2:55 PM

Keshav Jasrotia (003)

Mayukhmali Das

2:56 PM

Roll 98 Mayukhmali Das

Debadri Sengupta 001910701045

Rahul Saha 01910701009 Rahul Saha

Akansha Mukherjee

arpit mishra Roll number - 001910701092

Sarit Roy Chaudhury, 001910701096

Arpit Mishra - 001910701092

Mayukhmali Das Roll - 001910701098

001910701045 Debadri Sengupta

Akansha Mukherjee 001910701094

Rahul Saha

2:58 PM

001910701009 Rahul Saha

arpit mishra

2:59 PM

Arpit mishra 001910701092

Abhijit Deogharia

2:59 PM

001910701024 abhijit deogharia

Arijit Saha

2:55 PM

Arijit Saha Roll 19

TANMOY HALDER 1054

Roudrini Mukherjee Roll No. 001910701053

41 sagar sarkar

Umar Faruk Sarkar

2:55 PM

Umar Faruk Sarkar - 78

Arunim Ray

2:55 PM

Arunim Ray roll-42

Mayukhmali Das

2:55 PM

Mam , Mik not workingroll 98 001910701098

Utso Majumder

2:55 PM

Utso Majumder - Roll No 70

Rishav Basu

2:55 PM

Rishav Basu 40

Sagnik Das

2:55 PM

Sagnik Das 75

Abhishek Sarkar

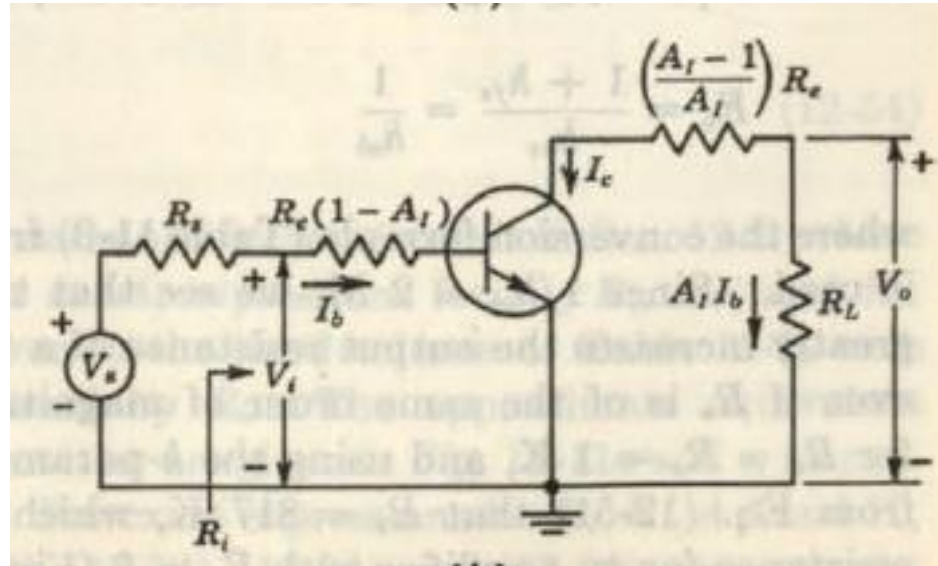
2:55 PM

Abhishek Sarkar- 27

Shubham singh

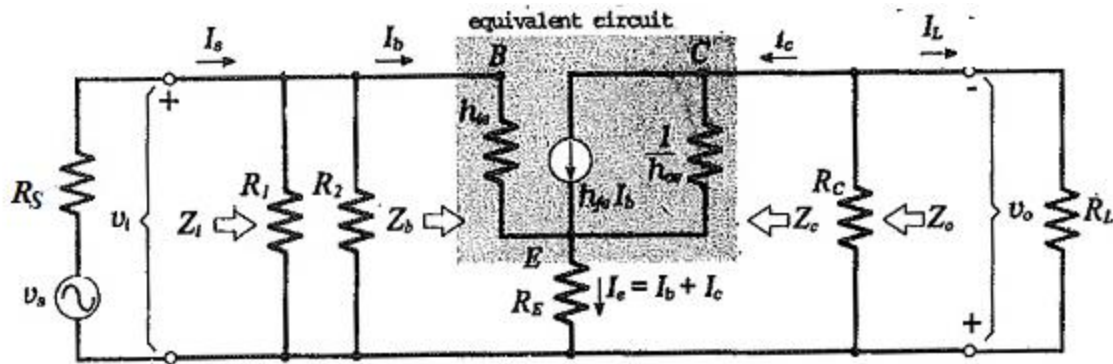
2:55 PM

Roll number 8 - Shubham Kumar Singh



$$A_I = \frac{-h_{fe}}{1 + h_{oe}R'_L} = \frac{-h_{fe}}{1 + h_{oe}\left(R_L + \frac{A_I - 1}{A_I}R_e\right)}$$

$$A_I = \frac{h_{oe}R_e - h_{fe}}{1 + h_{oe}(R_L + R_e)}$$



$$(R_s' + h_{ie})/h_{fe}$$

$$\begin{aligned} v_i &= I_b h_{ie} + I_e R_E \\ &= I_b h_{ie} + R_E (I_b + I_c) \\ &= I_b h_{ie} + R_E (I_b + h_{fe} I_b) \\ &= I_b [h_{ie} + R_E (1 + h_{fe})] \end{aligned}$$

$$Z_b = \frac{v_i}{I_b} = \frac{I_b [h_{ie} + R_E (1 + h_{fe})]}{I_b}$$

$$Z_b = h_{ie} + R_E (1 + h_{fe})$$

$$Z_i = R_1 \parallel R_2 \parallel Z_b$$

Including $R_s \rightarrow Z_{is} = R_s + Z_i$

$$\begin{aligned} v_o &= i_L R_L \\ i_L &= -h_{fe} i_b - h_{oe} v_{ce} - v_o / R_C \\ &= -h_{fe} i_b - h_{oe} (v_o - i_e R_E) - v_o / R_C \\ v_o &= (-h_{fe} i_b - h_{oe} (v_o - i_e R_E) - v_o / R_C) R_L \\ v_o (1 + h_{oe} R_L + R_L / R_C) &= -h_{fe} i_b R_L + h_{oe} (1 + h_{fe}) i_b R_E R_L \\ &= [-h_{fe} R_L + h_{oe} (1 + h_{fe}) R_E R_L] v_i / [(h_{ie} + R_E (1 + h_{fe}))] \end{aligned}$$

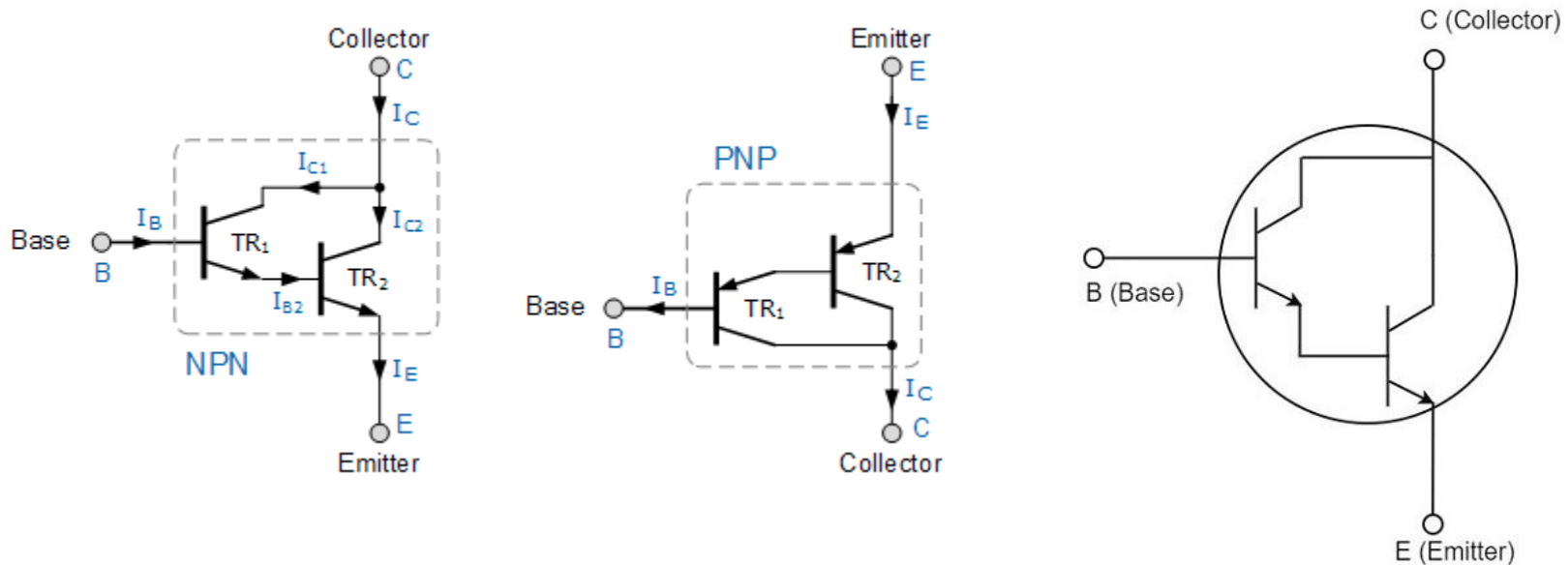
$$A_v = v_o / v_i = [-h_{fe} R_L + h_{oe} (1 + h_{fe}) R_E R_L] / [(h_{ie} + R_E (1 + h_{fe}))] * [(1 + h_{oe} R_L + R_L / R_C)]$$

$$A_v = v_o / v_i = \frac{[-h_{fe} + h_{oe} (1 + h_{fe}) R_E] R_L}{[(h_{ie} + R_E (1 + h_{fe}))] [(1 + h_{oe} R_L + R_L / R_C)]}$$

$$= \frac{[-h_{fe} + h_{oe} (1 + h_{fe}) R_E] R_L \parallel R_C \parallel 1/h_{oe}}{[(h_{ie} + R_E (1 + h_{fe}))]}$$

$$R_o = \frac{1}{h_{oe}} \frac{(1 + h_{fe}) R_E + (R_s + h_{ie})(1 + h_{oe} R_E)}{R_E + R_s + h_{ie} - h_{re} h_{fe} / h_{oe}}$$

The **Darlington Transistor** named after its inventor, Sidney Darlington is a special arrangement of two standard NPN or PNP bipolar junction transistors (BJT) connected together. The Emitter of one transistor is connected to the Base of the other to produce a more sensitive transistor with a much larger current gain being useful in applications where current amplification or switching is required.



Darlington Transistor pairs can be made from two individually connected bipolar transistors or a one single device commercially made in a single package with the standard: Base, Emitter and Collector connecting leads and are available in a wide variety of case styles and voltage (and current) ratings in both NPN and PNP versions.

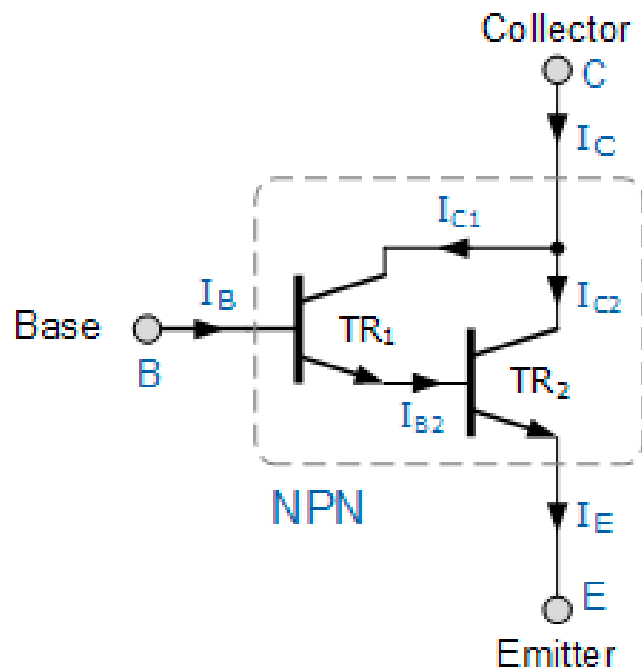
Characteristics

The following are the important characteristics of Darling ton amplifier.

Extremely high input impedance ($M\Omega$).

Extremely high current gain (several thousands).

Extremely low output impedance (a few Ω).



TR₁ drives the base of TR₂

$$I_C = I_{C1} + I_{C2}$$

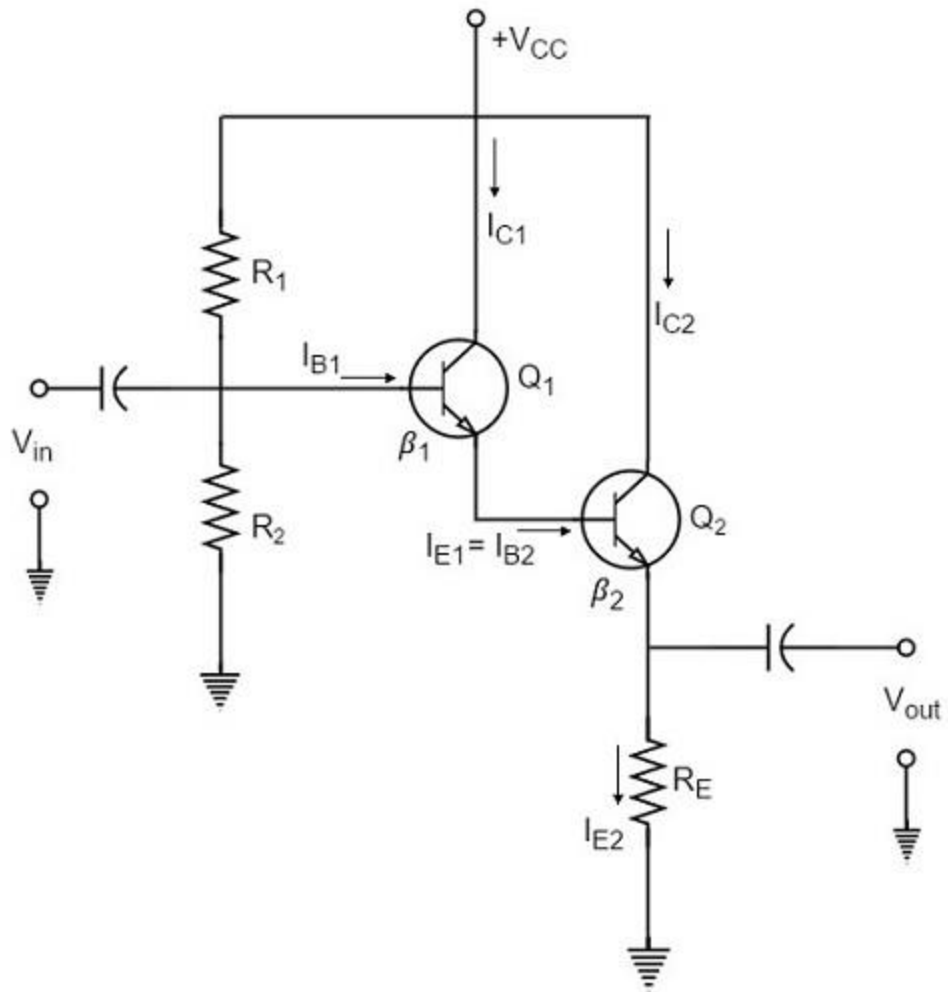
$$I_C = \beta_1 \cdot I_B + \beta_2 \cdot I_{B2}$$

$$I_{B2} = I_{E1} = I_{C1} + I_B = \beta_1 \cdot I_B + I_B = (\beta_1 + 1) \cdot I_B$$

$$I_C = \beta_1 \cdot I_B + \beta_2 \cdot (\beta_1 + 1) \cdot I_B$$

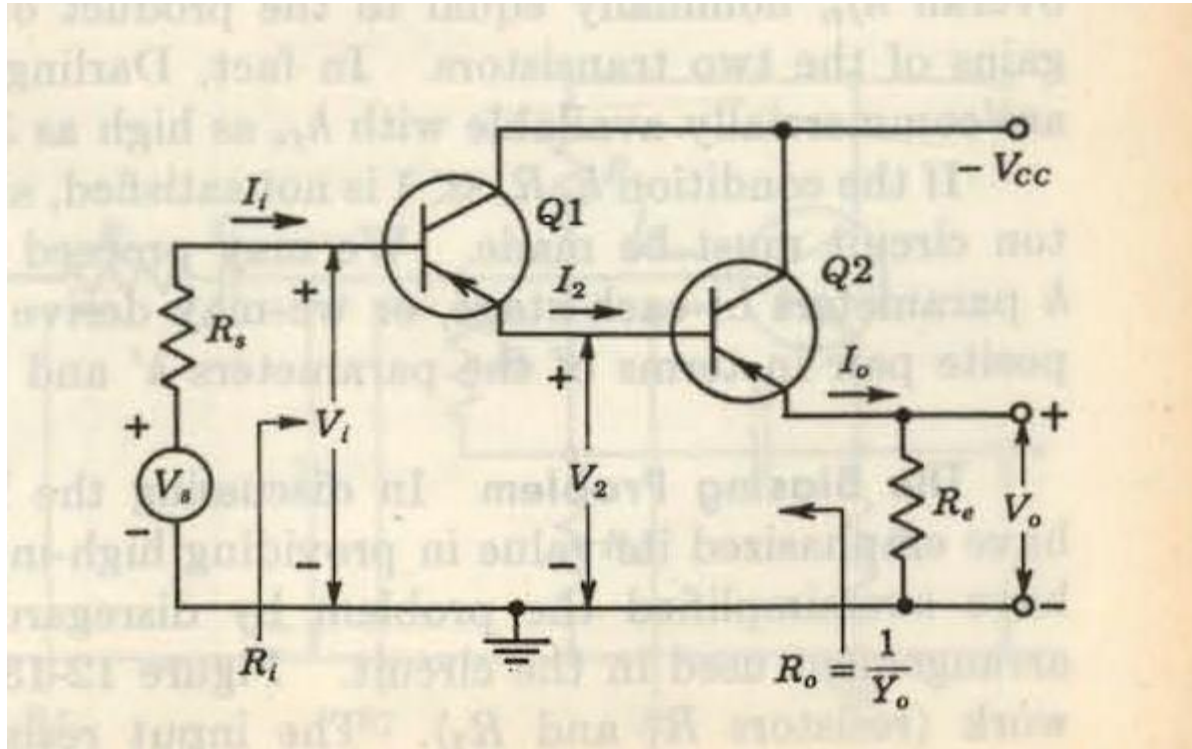
$$I_C = \beta_1 \cdot I_B + \beta_2 \cdot \beta_1 \cdot I_B + \beta_2 \cdot I_B$$

$$I_C = (\beta_1 + (\beta_2 \cdot \beta_1) + \beta_2) \cdot I_B$$



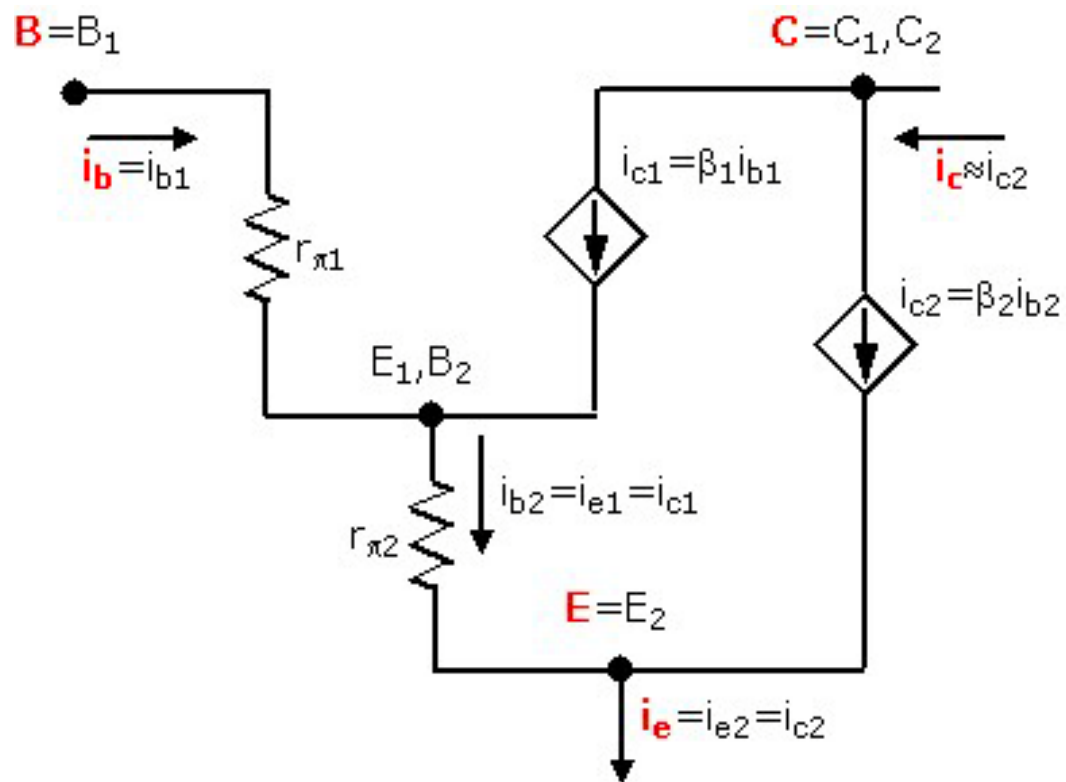
$$h_{ie1} + h_{ie2}\beta_2 + \beta_1\beta_2R_E$$

$$A_V = A_{V1}A_{V2} = \left(1 - \frac{h_{ie}}{R_{i2}}\right) \left(1 - \frac{h_{ie}}{A_{I1}R_{i2}}\right) \approx 1 - \frac{h_{ie}}{A_{I1}R_{i2}} - \frac{h_{ie}}{R_{i2}}$$

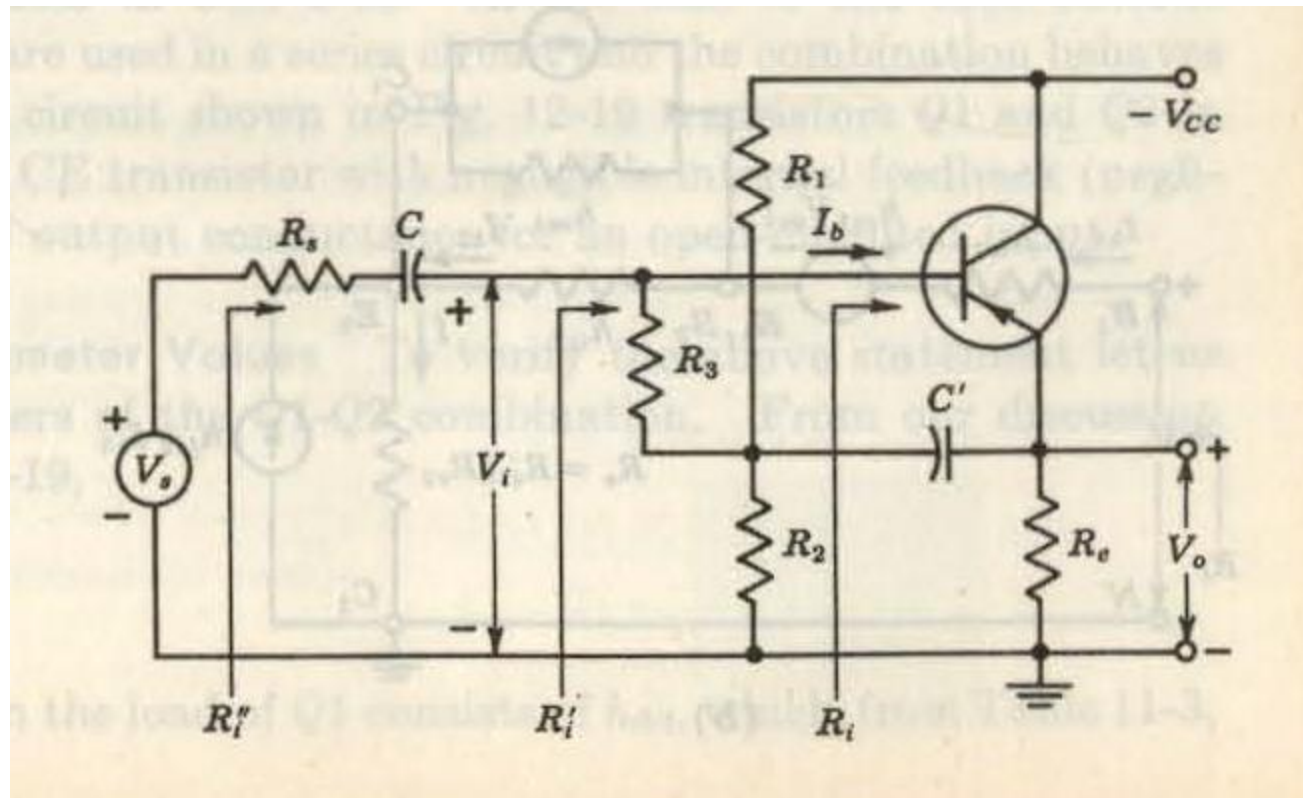


$$R_{i1} = h_{ie} + A_{I1}R_{i2} \approx \frac{(1 + h_{fe})^2 R_e}{1 + h_{oe}h_{fe}R_e}$$

$$R_{o2} \approx \frac{\frac{R_s + h_{ie}}{1 + h_{fe}} + h_{ie}}{1 + h_{fe}} = \frac{R_s + h_{ie}}{(1 + h_{fe})^2} + \frac{h_{ie}}{1 + h_{fe}}$$



$$\begin{aligned}
 R_{in} &= R_B \parallel [\beta_1 r_{\pi 2} + \beta_1 \beta_2 (R_E \parallel R_L)] \\
 &= R_B \parallel [\beta_1 \beta_2 (r_{e2} + (R_E \parallel R_L))]
 \end{aligned}$$



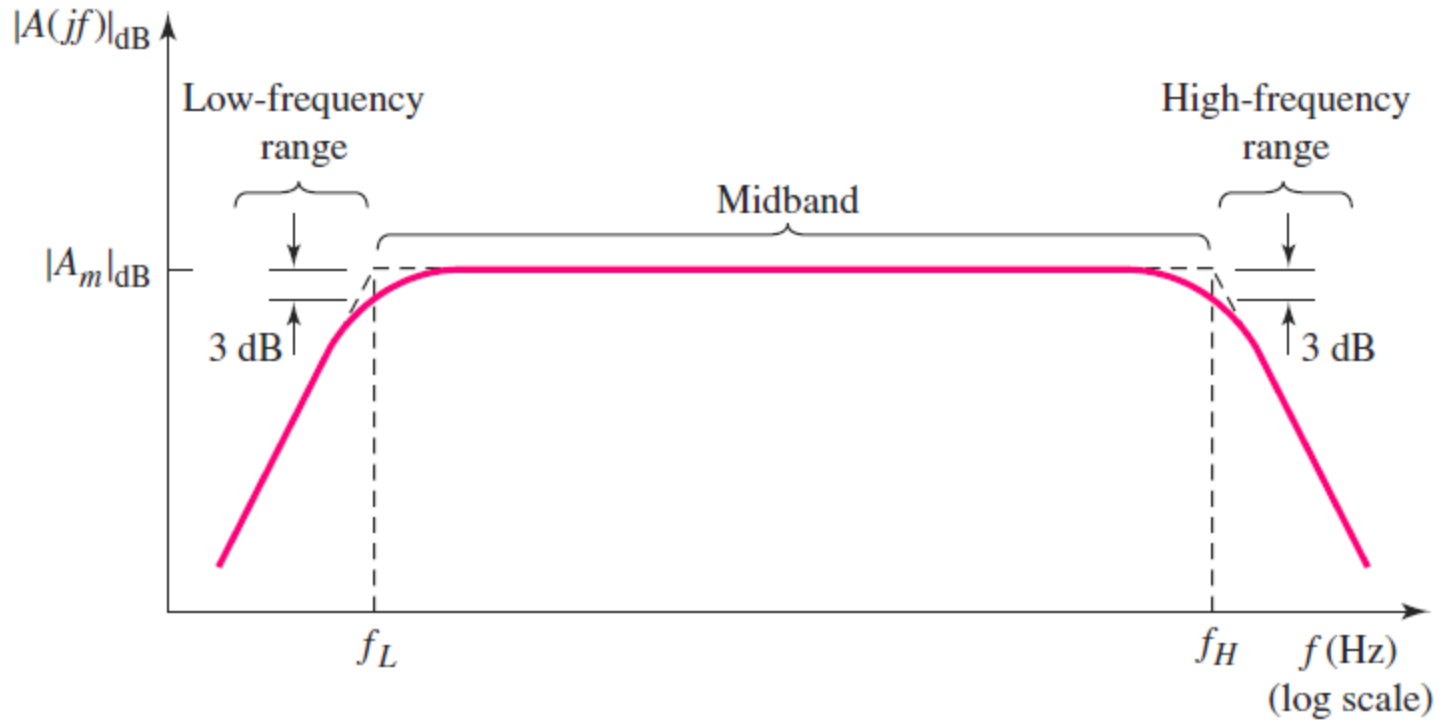
$$R_{\text{eff}} = \frac{R_3}{1 - A_V}$$

Multistage Amplifier

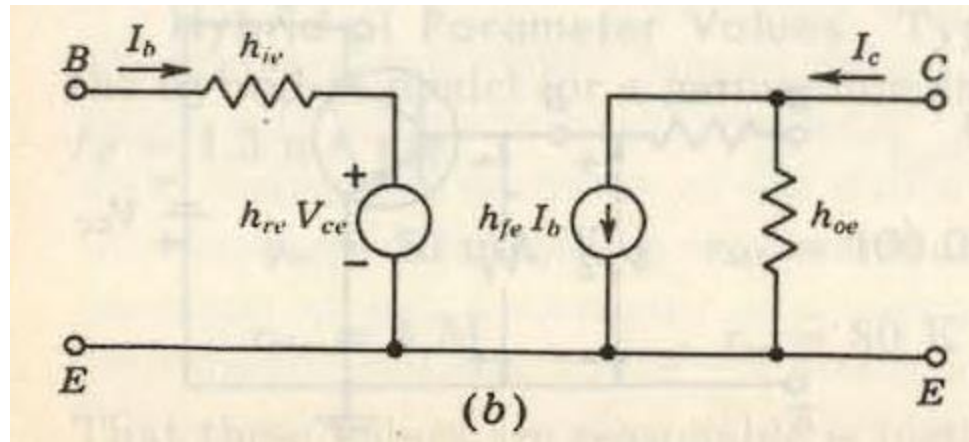
Cascaded

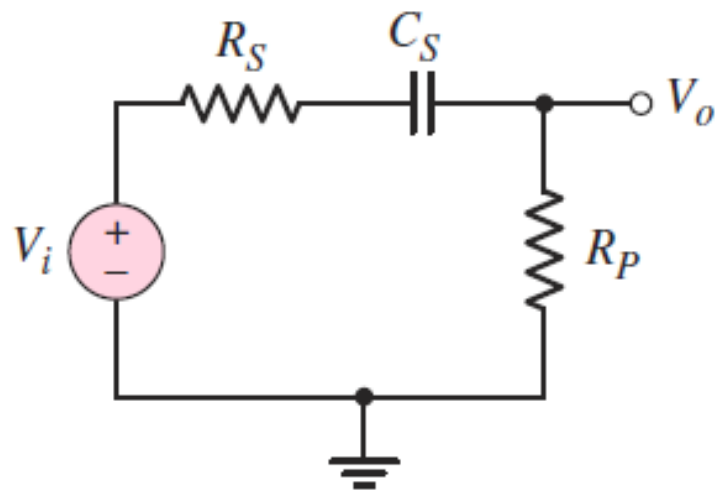
$$AV = Av_1 Av_2 \dots Av_n$$

$$A_v dB = 20 \log A_v$$



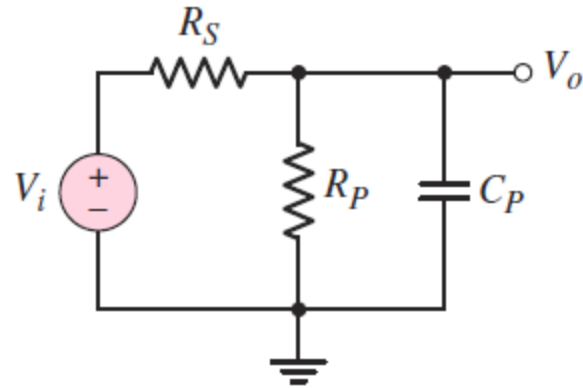
$1/wC \rightarrow$ open





$$\frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_S + R_P + \frac{1}{sC_S}}$$

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P} \right) \left[\frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S} \right] = K \left(\frac{s\tau_s}{1 + s\tau_s} \right)$$



$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P} \right) \left[\frac{1}{1 + s(R_S \parallel R_P)C_P} \right] = K \left(\frac{1}{1 + s\tau_P} \right)$$

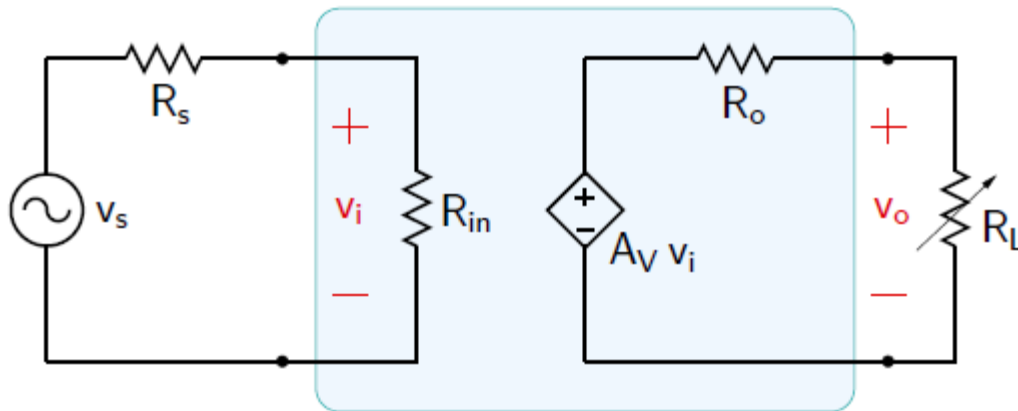
$-RC/(RE+re)$

$\theta = -\arctan(\omega/\omega_h)$

$$R_S = 1 \text{ k}\Omega,$$

$$R_P = 10 \text{ k}\Omega, C_S = 1 \text{ }\mu\text{F}, \text{ and } C_P = 3 \text{ pF}.$$

Determine the corner frequencies



Souvik Barman

2:28 PM

001910701076 076

Roudrini Mukherjee

2:28 PM

001910701053

Ayantik Das

2:28 PM

Ayantik Das Roll 089

INDRANIL Pal

2:28 PM

001910701057

Souvik Das

2:28 PM

001910701010

sagar sarkar

2:28 PM

041

sayak mandal

2:28 PM

001910701014

Keshav Jasrotia

2:28 PM

001910701003

Mayukhmali Das

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P} \right) \left[\frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S} \right] = K \left(\frac{s\tau_s}{1 + s\tau_s} \right) =$$

$$\tau_s = (R_S + R_P)C_S$$

$$|A_V| = K \frac{1}{\sqrt{1 + (1/\omega\tau_s)^2}} = K \frac{1}{\sqrt{1 + (\omega_l/\omega)^2}}$$

$$\omega_l = \frac{1}{(R_S + R_P)C_S}$$

$$\theta = \tan^{-1}(1/\omega\tau_s) = \tan^{-1}(\omega_l/\omega)$$

$$\text{At } \omega = \omega_l \quad \theta = 45^\circ$$