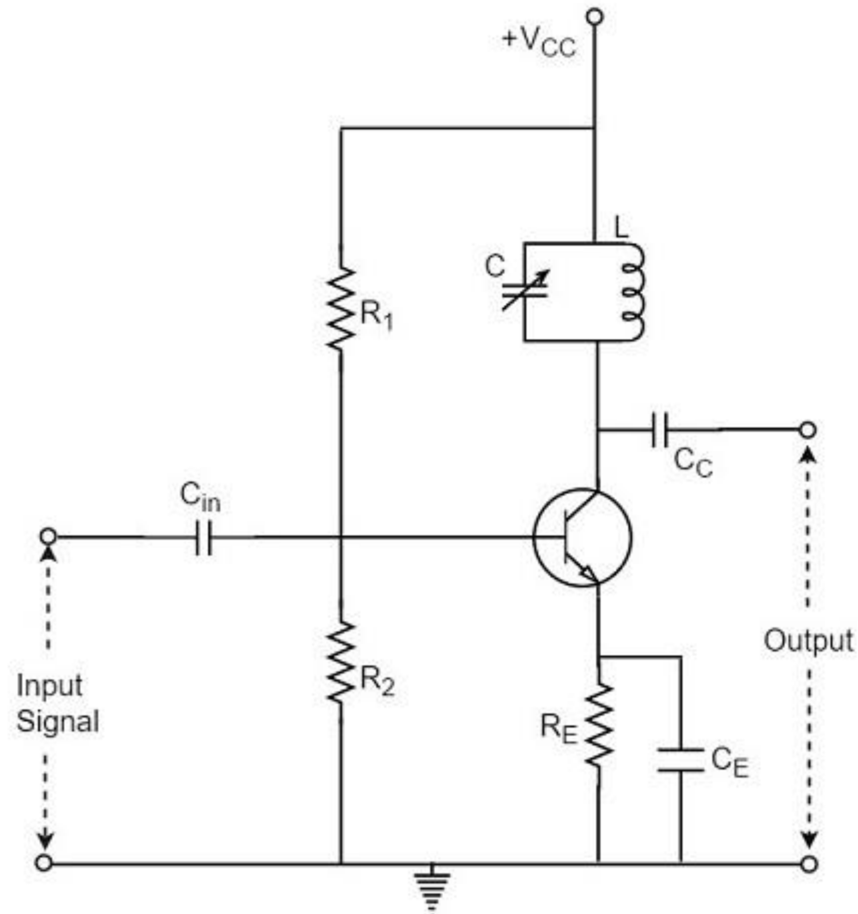
TUNED AMPLIFIERS

CM

Classification of tuned amplifiers

- Single tuned amplifier
- Double tuned amplifier
- Stagger tuned amplifier



Circuit diagram of single tuned amplifier

Operation

- The high frequency signal that has to be amplified is applied at the input of the amplifier.
- The resonant frequency of the parallel tuned circuit is made equal to the frequency of the signal applied by altering the capacitance value of the capacitor C, in the tuned circuit.
- At this stage, the tuned circuit offers high impedance to the signal frequency, which helps to offer high output across the tuned circuit.
- As high impedance is offered only for the tuned frequency, all the other frequencies which get lower impedance are rejected by the tuned circuit.
- Hence the tuned amplifier selects and amplifies the desired frequency signal. At resonant frequency f_r the impedance of parallel tuned circuit is very high and is purely resistive.
- The voltage across R_L is therefore maximum, when the circuit is tuned to resonant frequency.
- Hence the voltage gain is maximum at resonant frequency and drops off above and below it. The higher the Q, the narrower will the curve be.

Radio Frequency Spectrum: Ranges

Designation	Abbreviation	Frequencies	Wavelengths
Very Low Frequency	VLF	3 kHz - 30 kHz	100 km - 10 km
Low Frequency	LF	30 kHz - 300 kHz	10 km - 1 km
Medium Frequency	MF	300 kHz - 3 MHz	1 km - 100 m
High Frequency	HF	3 MHz - 30 MHz	100 m - 10 m
Very High Frequency	VHF	30 MHz - 300 MHz	10 m - 1 m
Ultra High Frequency	UHF	300 MHz - 3 GHz	1 m - 100 mm
Super High Frequency	SHF	3 GHz - 30 GHz	100 mm - 10 mm
Extremely High Frequency	EHF	30 GHz - 300 GHz	10 mm - 1 mm

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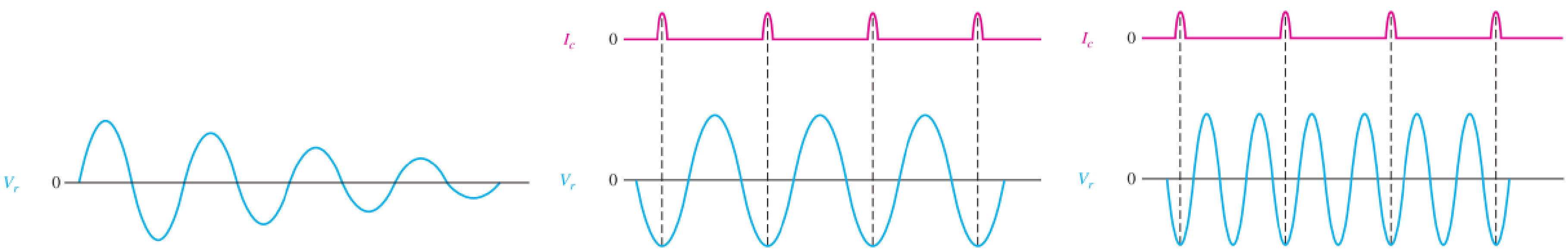


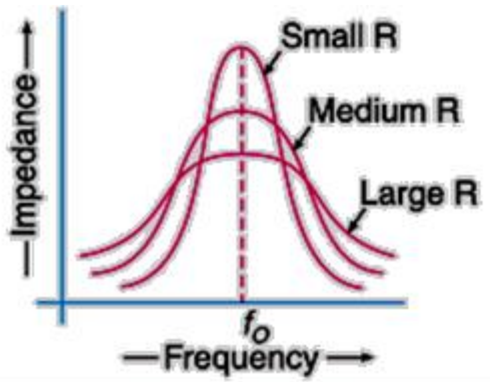
(a) An oscillation will gradually die out (decay) due to energy loss. The rate of decay depends on the efficiency of the tank circuit.

(b) Oscillation at the fundamental frequency can be sustained by short pulses of collector current.

(c) Oscillation at the second harmonic frequency

Quality factor

- The resonance curve is required to be as sharp as possible in order to provide a high selectivity.
- A sharp resonance curve means that the impedance falls off rapidly as the frequency is varied above and below the resonant frequency.



Effect of coil resistance (*R*) on the sharpness of resonance curves.

the *Q-factor*, $Q_o = \dfrac{X_L}{R} = \dfrac{\omega_o \cdot L}{R} = \dfrac{2\pi f_o \cdot L}{R}$

L = Value of circuit inductance, and

R = Value of circuit resistance or coil resistance.

$$\text{BW} = \frac{f_o}{Q_o}$$

$$f_o = BW \times Q_o$$

- A higher value of quality factor (Qo) provides a higher selectivity but a smaller bandwidth and vice versa.

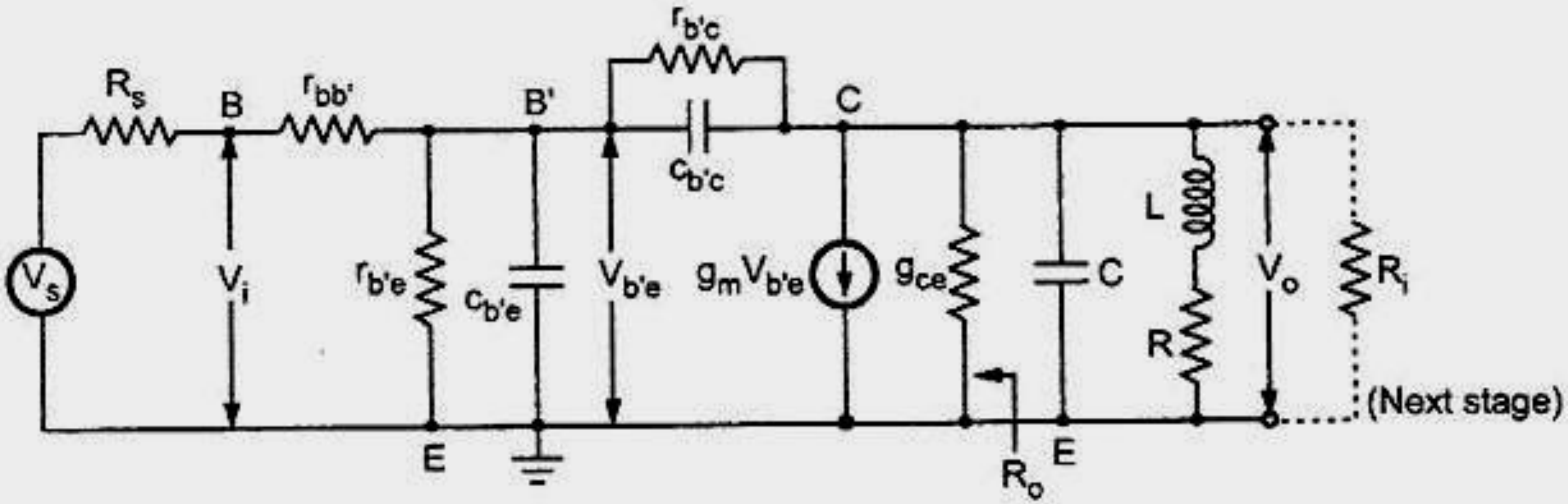


Fig. 3. Equivalent circuit of single tuned amplifier

The Fig. 3. shows the equivalent circuit for single tuned amplifier using hybrid π parameters.

As shown in the Fig. 3 R_i is the input resistance of the next stage and R_o is the output resistance of the current generator $g_m V_{b'c}$. The reactances of the bypass capacitor C_E and the coupling capacitors C_C are negligibly small at the operating frequency and hence these elements are neglected in the equivalent circuit shown in the Fig. 3.

The equivalent circuit shown in Fig. 3. can be simplified by applying Miller's theorem. Fig. 4 shows the simplified equivalent circuit for single tuned amplifier.

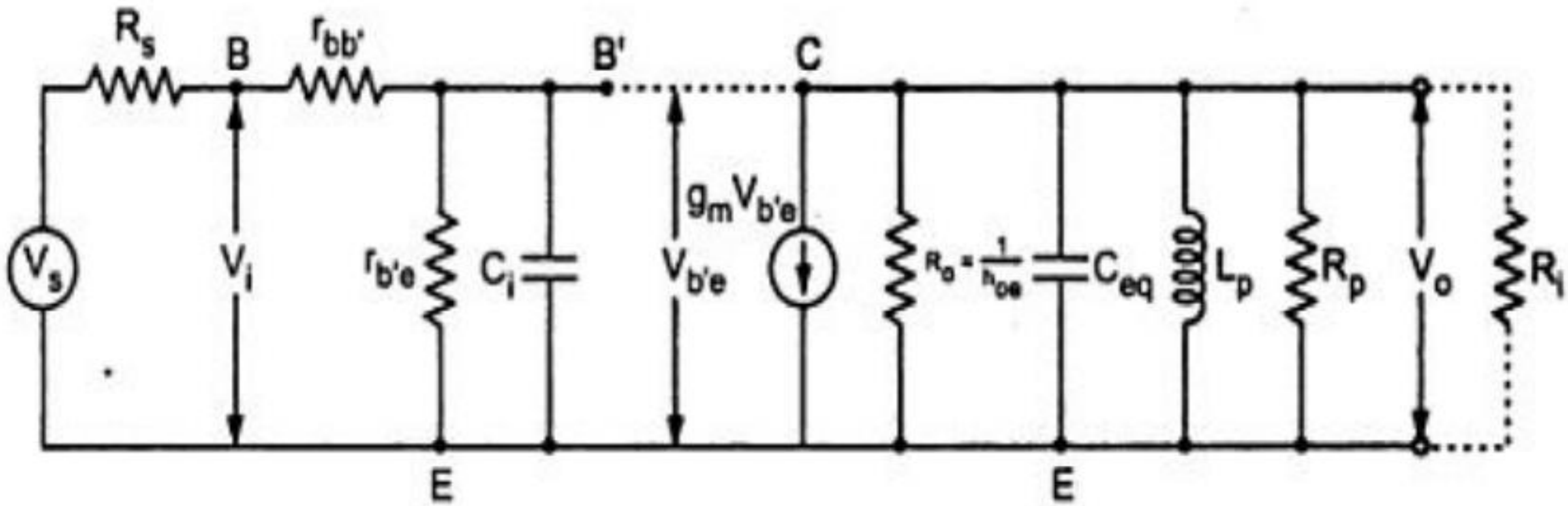


Fig. 4

Ci – input capacitance and Ceq – output capacitance,

$C_i = C_{b'e} + C_{b'c}(1 - A)$ where A is the voltage gain of the amplifier. (1)

$C_{eq} = C_{b'c}\left(\frac{A-1}{A}\right) + C$ where C is the tuned circuit capacitance. (2)

Ro=1/hoe (3)

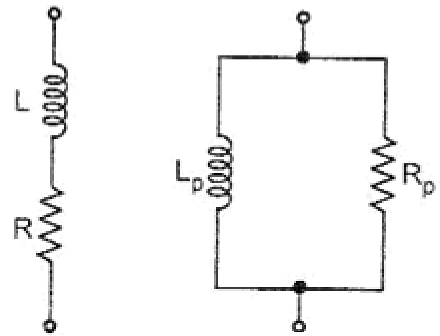


Fig. 5

The series RL circuit is represented by its equivalent parallel circuit. The conditions for equivalence are most easily established by equating the admittances of the two circuits shown in Fig 5

Admittance of the series combination of RL is given as,

$$Y = \frac{1}{R + j\omega L}$$

Multiplying numerator and denominator by $R - j\omega L$ we get,

$$Y = \frac{R - j\omega L}{R^2 + \omega^2 L^2} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega^2 L}{\omega(R^2 + \omega^2 L^2)}$$

$$= \frac{1}{R_p} + \frac{1}{j\omega L_p}$$

where $$R_p = \frac{R^2 + \omega^2 L^2}{R} \qquad \dots (4)$$

and $$L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} \qquad \dots (5)$$

Centre frequency

The centre frequency or resonant frequency is given as,

$$f_r = \frac{1}{2\pi\sqrt{L_p C_{eq}}} \quad \ldots(6)$$

where $L_p = \dfrac{R^2 + \omega^2 L^2}{\omega^2 L}$

and $$C_{eq} = C_{b'c}\left(\frac{A-1}{A}\right) + C \quad \ldots(7)$$

$$= C_o + C$$

Therefore, C_{eq} is the summation of transistor output capacitance and the tuned circuit capacitance.

Quality factor Q

The quality factor Q of the coil at resonance is given by,

$$Q_r = \frac{\omega_r L}{R} \quad \ldots(8)$$

This quality factor is also called unloaded Q. but in practice, transistor output resistance and input resistance of next stage act as a load for the tuned circuit. The quality factor including load is called as loaded Q and it can be given as follows:

From equation (4) we have,

$$R_p = \frac{R^2 + \omega^2 L^2}{R} = R + \frac{\omega^2 L^2}{R}$$

As $\frac{\omega^2 L^2}{R} >> 1$, $R_p \approx \frac{\omega^2 L^2}{R}$ ...(9)

From equation (5) we have,

$$L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} = \frac{R^2}{\omega^2 L} + L$$

$$\approx L \qquad \because \omega L >> R \quad \text{... (10)}$$

From equation (9), we can express R_p at resonance as,

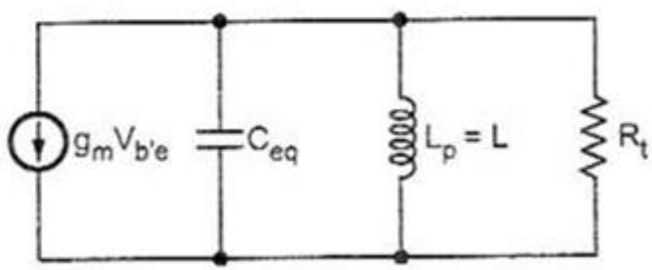
$$R_p = \frac{\omega_r^2 L^2}{R}$$

$$= \omega_r Q_r L \quad \because Q_r = \frac{\omega_r L}{R} \quad \text{... (11)}$$

Therefore, Q_r can be expressed interms of R_p as,

$$Q_r = \frac{R_p}{\omega_r L} \quad \text{...(12)}$$

The effective quality factor including load can be calculated looking at the simplified equivalent output circuit for single tuned amplifier.



where $R_t = R_o \parallel R_p \parallel R_i$

Fig. 6 **Simplified output circuit for single tuned amplifier**

$$\text{Effective quality factor } Q_{eff} = \frac{\text{Susceptance of inductance L or capacitance C}}{\text{Conductance of shunt resistance } R_t}$$

$$= \frac{R_t}{\omega_r L} \text{ or } \omega_r C_{eq} R_t \qquad \ldots (13)$$

$$A_v = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \times \frac{R_t}{1 + j(\omega R_t C - R_t/\omega L)} = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \times \frac{R_t}{1 + 2jQ_{eff}\delta}$$

where $R_t = R_o || R_p || R_i$

δ = Fraction variation in the resonant frequency

$$A_v \text{ (at resonance)} = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \times R_t$$

$$\therefore \left| \frac{A_v}{A_v(\text{at resonance})} \right| = \frac{1}{\sqrt{1 + (2\delta Q_{eff})^2}}$$

3 dB bandwidth

The 3 dB bandwidth of a single tuned amplifier is given by,

$$\Delta f = \frac{1}{2\pi R_t C_{eq}}$$

$$= \frac{\omega_r}{2\pi Q_{eff}} \quad \because Q_{eff} = \omega_r R_t C_{eq}$$

$$= \frac{f_r}{Q_{eff}} \quad \because \omega_r = 2\pi f_r$$

Example 3.1 : *Design a single tuned amplifier for following specifications :*

1. *Centre frequency = 500 kHz*
2. *Bandwidth = 10 kHz*

Assume transistor parameters : $g_m = 0.04$ *S,* $h_{fe} = 100$, $C_{b'e} = 1000$ *pF and* $C_{b'c} = 100$ *pF. The bias network and the input resistance are adjusted so that* $r_i = 4$ *kΩ and* $R_L = 510\ \Omega$.

Solution : From equation (9) we have,

$$BW = \frac{1}{2\pi RC}$$

$$\therefore \quad RC = \frac{1}{2\pi BW} = \frac{1}{2\pi \times 10 \times 10^3}$$

$$= 15.912 \times 10^{-6}$$

From equation (3) we have,

$$R = r_i \parallel R_p \parallel r_{b'e}$$

where

$$r_i = 4\ k\Omega$$

$$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{100}{0.04} = 2500\ \Omega$$

$$R_p = Q_c\, \omega_o L = \frac{Q_c}{\omega_o C}$$

$$\therefore \quad R = 4 \times 10^3 \parallel 2500 \parallel \frac{Q_c}{\omega_o C}$$

$$C = \frac{1}{2\pi \times 10 \times 10^3 \times R}$$

$$\therefore \quad C = \frac{1}{2\pi \times 10 \times 10^3 \times \left[4 \times 10^3 \,||\, 2500 \,||\, \dfrac{Q_c}{2\pi \times 500 \times 10^3 \times C}\right]}$$

The typical range for Q_c is 10 to 150. However, we have to assume Q such that value of C_p should be positive. Let us assume Q = 100.

$$\therefore \quad C = \frac{1}{2\pi \times 10 \times 10^3 \left[1538.5 \,||\, \dfrac{1}{2\pi \times 5000 \times C}\right]}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \left[\dfrac{1}{\dfrac{1}{1538.5} + 2\pi \times 5000 \times C}\right]}$$

Solving for C we get,

$$C = 0.02\ \mu F$$

We have,

$$C = C' + C_{b'e} + (1 + g_m R_L)\, C_{b'c}$$

$$\therefore \quad C' = C - [C_{b'e} + (1 + g_m R_L)\, C_{b'c}]$$

$$= 0.02 \times 10^{-6} - [1000 \times 10^{-12} + (1 + 0.04 \times 510) \times 100 \times 10^{-12}]$$

$$\therefore \quad C' = \mathbf{0.01686\ \mu F}$$

We have,

$$\omega_o^2 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(2\pi \times 500 \times 10^3)^2 \times 0.02 \times 10^{-6}}$$

$$= \mathbf{5\ \mu H}$$

From equation (2) we have,

$$R_p = \omega L Q_c = 2\pi \times 500 \times 10^3 \times 5 \times 10^{-6} \times 100$$

$$= \mathbf{1570\ \Omega}$$

$$\therefore \quad R = r_i \parallel R_p \parallel r_{b'e}$$

$$= 4 \times 10^3 \parallel 1570 \parallel 2500$$

$$= 777\ \Omega$$

We have mid frequency gain as,

$$A_{i\,max} = -g_m R = (-0.04)(777) = \mathbf{-31}$$

LIMITATION

- This tuned amplifier are required to be highly selective. But high selectivity required a tuned circuit with a high Q-factor .
- A high Q- factor circuit will give a high A_v but at the same time , it will give much reduced band with because bandwidth is inversely proportional to the Q- factor .
- It means that tuned amplifier with reduce bandwidth may not be able to amplify equally the complete band of signals & result is poor reproduction . This is called potential instability in tuned amplifier.

Double tuned amplifier

- Double tuned amplifier is one (or) more stages with each stage using coupled circuits having different frequencies of resonance. The two resonant circuits are normally inductively coupled. The tuning is done in both secondary and primary circuits. Following are some of the advantages of double tuned amplifiers:
  - a) **Large gain bandwidth :** For simplicity assume the stages are non-interacting (Generally this is the case as these stages are inductively coupled) then the
    - overall voltage Gain = $(Av)^n/(1+w^2)^{(1/2)}$ and bandwidth = $B.W(2^{(1/n)}-1)^{(1/2)}$ (since for n Non-interacting stages with each stage having lower and upper cut-off frequencies as fl, fh , the upper and lower cut-off frequencies are given as follows fh" = fh*(2^(1/n)-1)^(1/2) , fl" = fl/(2^(1/n)-1)^(1/2) and fh">>fl").
  - b) Large 3 db bandwidths.
  - c) Provides a frequency response having flatter sides.
  - d) Sensitivity can be increased due to increased overall gain. Here sensitivity refers to the ability to receive weak signals.
  - e) Selectivity can be increased. This is due to decrease in quality factor due to increase in bandwidth. Sensitivity is inversely proportional to quality factor. Selectivity refers to the ability to discriminate the signals in adjacent band.
  - Quality factor of a double tuned amplifier is given as
  - $$Qeff = fr(2^{(1/n)} -2)^{(1/2)}/(B.W)$$
  - Double tuned amplifier are used in many applications, one of which is IF amplifier in analog communication receiver.

Double tuned amplifier:

The below figure shows double tuned RF amplifier in CE configuration. Here, voltage developed across tuned circuit is coupled inductively to another tuned circuit. Both tuned circuits are tuned to the same frequency.

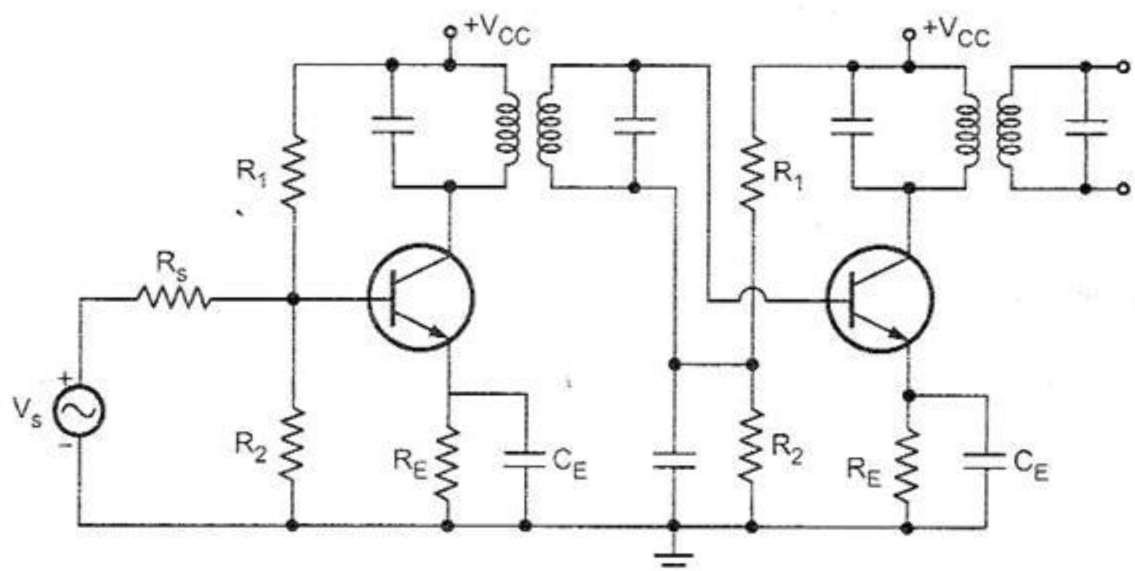
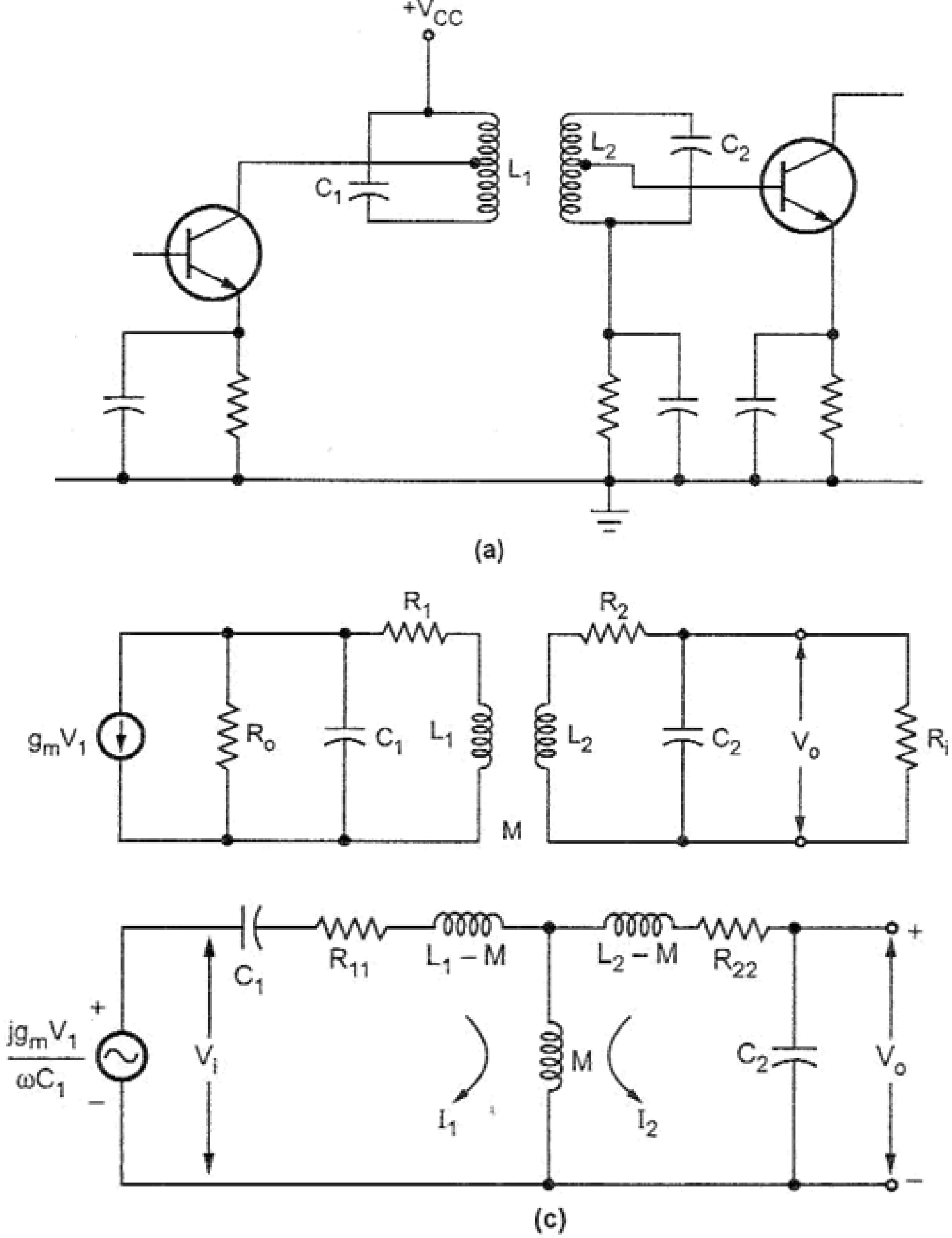


Fig. 7



Therefore we can write,

$$R_{11} = \frac{\omega_o^2 L_1^2}{R_o} + R_1$$

$$R_{12} = \frac{\omega_o^2 L_2^2}{R_i} + R_2$$

$$M = k\sqrt{L_1 L_2}$$

where, k is the coefficient of coupling.

(c)

Fig. 8 **Equivalent circuits for double tuned amplifier**

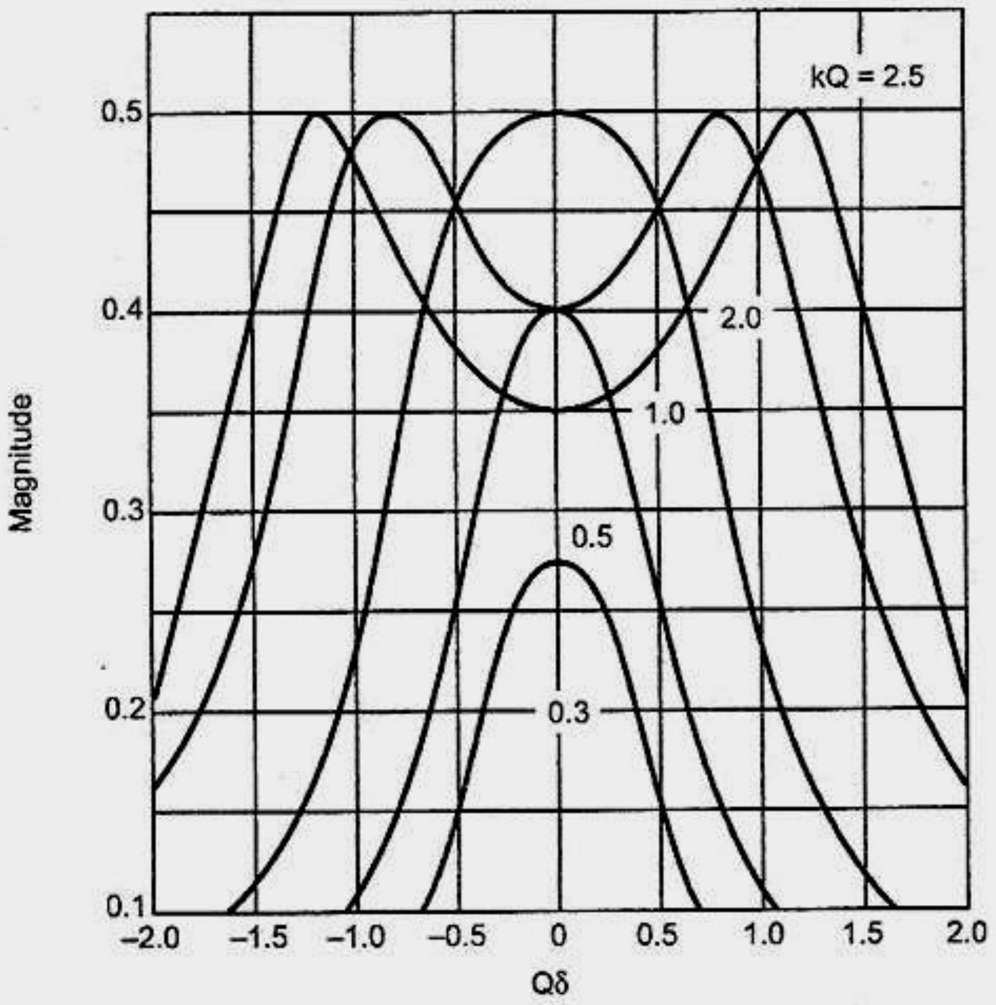
$$A_v = \frac{V_o}{V_i} = g_m \omega_r^2 L_1 L_2 \left[\frac{kQ^2}{\omega_r \sqrt{L_1 L_2}\left[4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2)\right]} \right]$$

$$|A_v| = g_m \omega_r \sqrt{L_1 L_2}\, Q \frac{kQ}{\sqrt{1 + k^2Q^2 - 4Q^2\delta^2 + 16Q^2\delta^2}}$$

The frequency deviation δ at which the gain peaks occur can be found by maximizing equation (4), i.e.

$$4Q\delta - j(1 + k^2Q^2 - 4Q^2\delta^2) = 0 \qquad \dots (6)$$

frequency response of double tuned amplifier



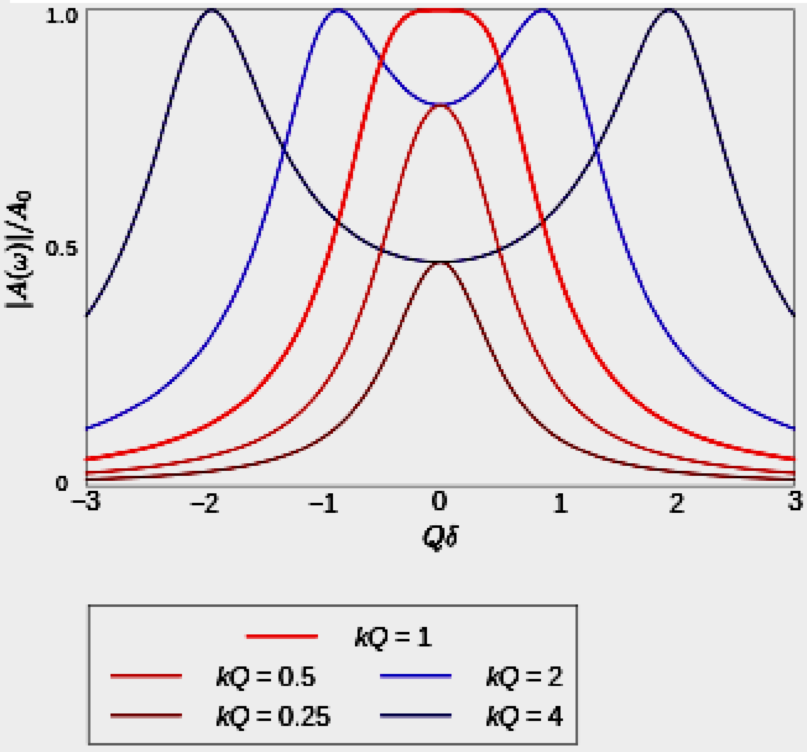


Fig. 9

- For $k = \frac{1}{Q}$, $f_1 = f_2 = f_r$. This condition is known as **critical coupling**.
- For $k < 1/Q$, the peak gain is less than maximum gain and the coupling is poor.
- For $k > 1/Q$, the circuit is overcoupled and the response shows the double peak.

double peak response is useful when more bandwidth is required.

The gain magnitude at peak is given as,

$$|A_p| = \frac{g_m \omega_o \sqrt{L_1 L_2}\, kQ}{2}$$

And gain at the dip at $\delta = 0$ is given as,

$$|A_d| = |A_p| \frac{2\,kQ}{1 + k^2Q^2}$$

The ratio of peak gain and dip gain is denoted as γ and it represents the magnitude of the ripple in the gain curve.

$$\gamma = \left|\frac{A_p}{A_d}\right| = \frac{1 + k^2Q^2}{2\,kQ}$$

Using quadratic simplification and choosing positive sign we get,

$$kQ = \gamma + \sqrt{\gamma^2 - 1}$$

The bandwidth between the frequencies at which the gain is $|A_d|$ is the useful bandwidth of the double tuned amplifier. It is given as,

$$BW = 2\,\delta' = \sqrt{2}\,(f_2 - f_1)$$

At 3 dB bandwidth,

$$\gamma = \sqrt{2}$$

$$\therefore \quad kQ = \gamma + \sqrt{\gamma^2 - 1} = \sqrt{2} + \sqrt{\sqrt{2}^2 - 1} = 2.414$$

$$\therefore \quad 3\text{ dB BW} = 2\,\delta' = \sqrt{2}\,(f_2 - f_1)$$

$$= \sqrt{2}\left[f_r\left(1 + \frac{1}{2Q}\sqrt{k^2Q^2 - 1}\right) - f_r\left(1 - \frac{1}{2Q}\sqrt{k^2Q^2 - 1}\right)\right]$$

$$= \sqrt{2}\left[\left(\frac{f_r}{Q}\sqrt{k^2Q^2 - 1}\right)\right]$$

$$= \sqrt{2}\left[\frac{f_r}{Q}\sqrt{(2.414)^2 - 1}\right] = \frac{3.1\,f_r}{Q}$$

We know that, the 3 dB bandwidth for single tuned amplifier is 2 f_r/Q. Therefore, the 3 dB bandwidth provided by double tuned amplifier ($3.1f_r/Q$) is substantially greater than the 3 dB bandwidth of single tuned amplifier.

Compared with a single tuned amplifier, the double tuned amplifier

1. Possesses a flatter response having steeper sides.
2. Provides larger 3 dB bandwidth.
3. Provides large gain-bandwidth product.

Effect of Cascading Single Tuned Amplifier on Bandwidth

Prove that Band width of n-stage single tuned amplifier is

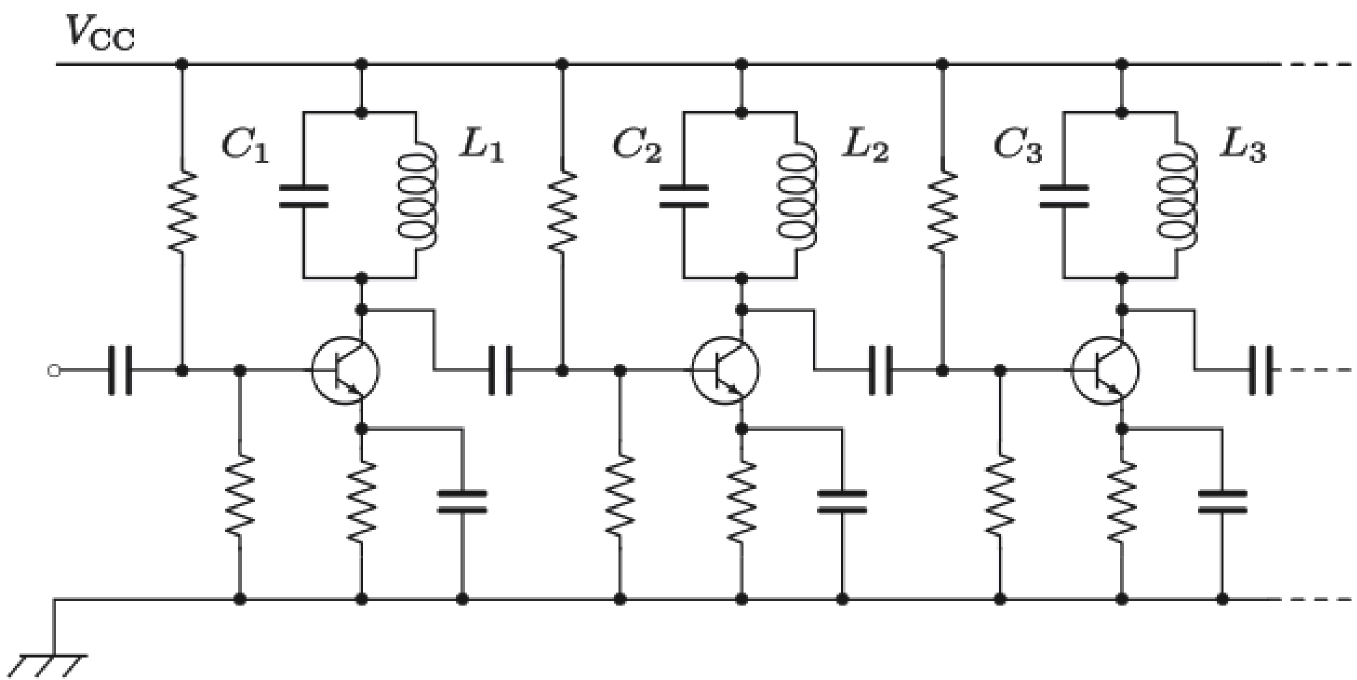
$$BW_n = BW_1 \sqrt{2^{\frac{1}{n}} - 1}$$

Stagger Tuned Amplifier

- The double tuned amplifier gives greater 3dB bandwidth having steeper sides and flat top.
- But alignment of double tuned amplifier is difficult.
- To overcome this problem
  - two single tuned cascaded amplifiers having certain bandwidth are taken and
  - Their resonant frequencies are so adjusted that they are separated by an amount equal to the bandwidth of each stage. Since resonant frequencies are displaced or staggered, they are known as stagger tuned amplifiers.

The advantage of stagger tuned amplifier is to have a

- better flat, wideband characteristics in contrast with a very sharp, selective, narrow band characteristics of synchronously tuned circuits (tuned to same resonant frequencies).
- Fig. below shows the relationship of amplification characteristics of individual stages in a staggered pair to the overall amplification of the two stages.



V_{CC}
C_1
L_1
C_2
L_2
C_3
L_3

STAGGER TUNED AMPLIFIERS

- It is a multistage amplifier which has one parallel resonant circuit for every stage, while resonant frequency of every stage

  is slightly different from previous stages.

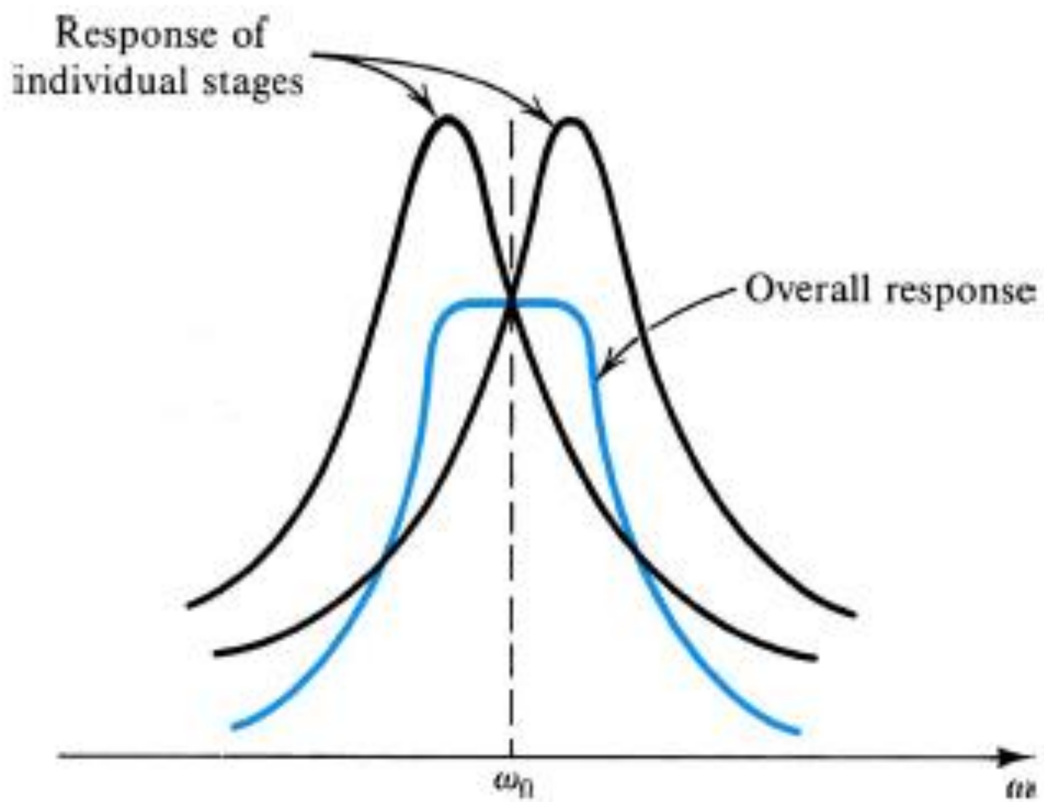
  From circuit diagram it is clear that first stage of this amplifiers has a resonant circuit formed by L_1 & C_1 that

  $$f_1 = 1 / (2\Pi \sqrt{L_1 C_1})$$

  The o/p of stage is applied to second stage which is tuned to slightly higher frequency.

  $$f_2 = 1 / (2\Pi \sqrt{L_2 C_2})$$

Stagger Tuned Amplifier



- Single tuned amplifier with separate resonant frequencies are used in stagger tuned amplifier.
- The resonant frequencies are

$$f_{r1}=f_r-\delta \text{ and } f_{r2}=f_r+\delta$$

The gain of single tuned amplifier is

$$\frac{A_v}{A_{vresonance}} = \frac{1}{1+2jQ_{eff}\delta} = \frac{1}{1+jX}$$

According to this tuned frequencies, the selectivity function is given as,

$$\left(\frac{A_v}{A_{vresonance}}\right)_1 = \frac{1}{1+j(X-1)}$$

$$\left(\frac{A_v}{A_{vresonance}}\right)_2 = \frac{1}{1+j(X+1)}$$

- The overall gain is product of individual gain of the 2 stages.

$$\left(\frac{A_v}{A_{vresonance}}\right)_{cascaded} = \frac{1}{2\sqrt{1+4Q_{eff}^4\delta^4}}$$

- At Qδ=0 overall normalized gain is 0.5.