

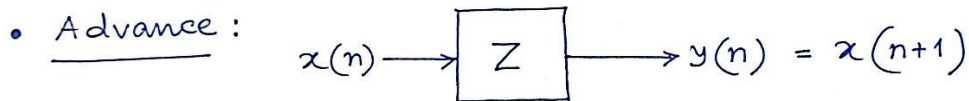
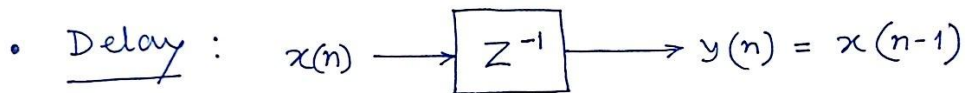
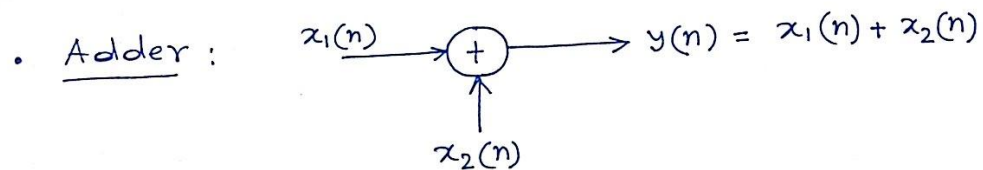
Digital Signal Processing

M. K. Naskar

Different representations

- Block diagram
- System functional
- Difference equation
- Impulse response

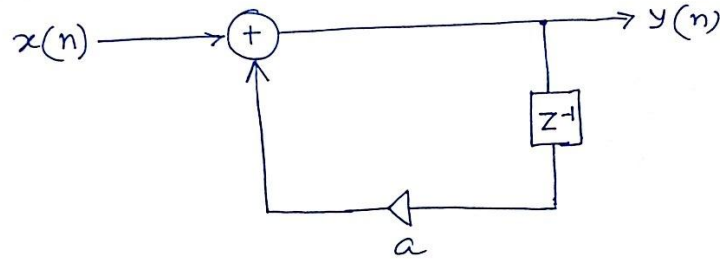
Elements of a Digital Filter



Characterization of Digital Filters

(Difference eqn. representation of a digital filter)

(Process of analysis)



Difference equation $\Rightarrow$

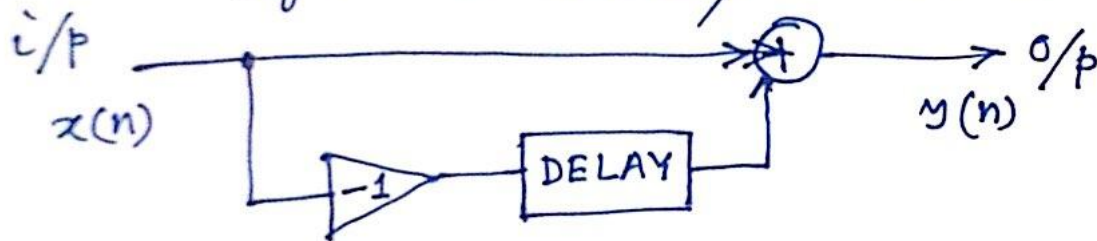
$$y(n) = x(n) + a y(n-1)$$

So, ~~$y(n) = G[x(n)] = x(n) + a y(n-1)$~~

Prob. Block diagram of a DT system is given

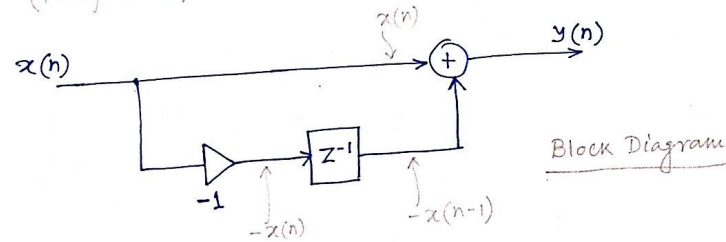
below. Sketch its unit-sample response.

Assume that system is initially at rest i.e. o/p of the DELAY is 0;



- Non-recursive
(Feedforward)
- 1st order

First-order Non-recursive Filter (Feedforward)



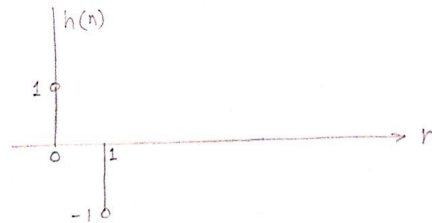
Formal Approach :

$$y(n) = x(n) - x(n-1] \leftarrow \text{Difference equation}$$

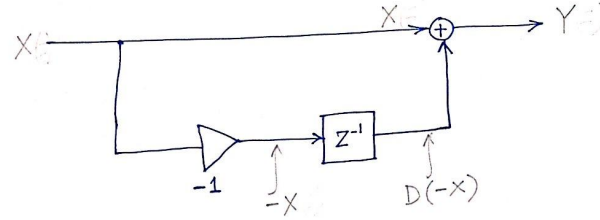
$$\Rightarrow h(n) = \delta(n) - \delta(n-1]$$

Impulse (unit sample) response.

Sketch of $h(n)$:



First-order Non-recursive Filter

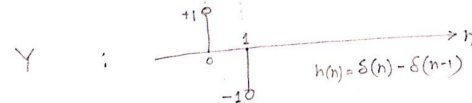
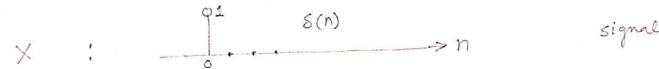


Operator approach : (signal as a whole)

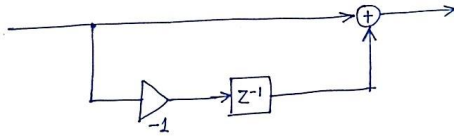
$$Y = X + D(-X)$$

$$\Rightarrow Y = X - DX = (1-D)X$$

↖ This gives the algorithm to obtain Y from X.

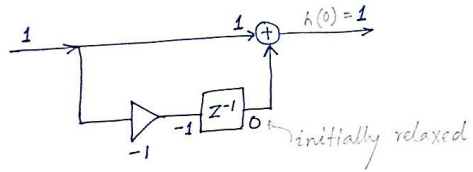


First-order Non-recursive Filter



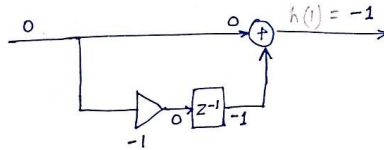
Informal Approach (Sample by sample analysis) (Intuitive)

At $n=0$,



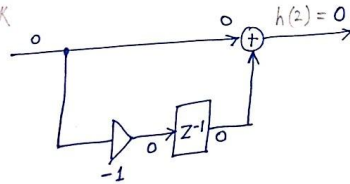
After clocking
↓

At $n=1$,



After another clock
↓

At $n=2$



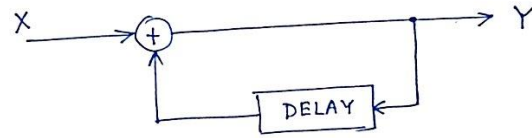
$$\text{Hence, } h(n) = \{ \underset{\uparrow}{1}, -1, 0, \dots \} = \delta(n) - \delta(n-1)$$

Prob. Block diagram of DT system is given below. Sketch the i/p sequence for which the o/p is a unit step sequence. Assume that the system is initially at rest.



- Recursive
- 1st order

First-order
Recursive Filter:
(Feedback)



First-order Filter

$$Y = X + DY \quad \leftarrow \text{using operators notation.}$$

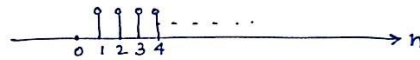
$$\Rightarrow Y - DY = X, \quad \text{i/p can be determined from its o/p.}$$

Example:

Y :



DY :



Right
Shift
(Delayed)

-DY :



Inverted

Y-DY :



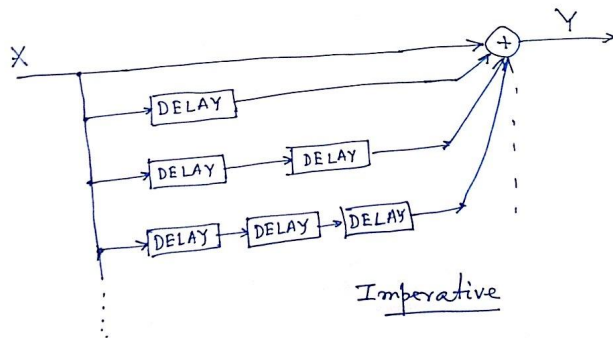
with sample
sequence



Not imperative

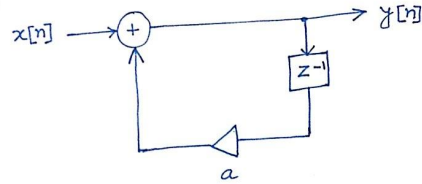
$$\begin{aligned}
 Y &= X + BY \\
 \Rightarrow Y - BY &= X \\
 \Rightarrow (1 - B)Y &= X \\
 \Rightarrow Y &= \left(\frac{1}{1 - B} \right) X \\
 &= (1 + B + B^2 + B^3 + \dots) X
 \end{aligned}$$

$$\begin{array}{r}
 (1 - B) \frac{1}{1 - B} (1 + B + B^2 + \dots) \\
 \hline
 B - B^2 \\
 \hline
 B^2 - B^3 \\
 \hline
 B^3
 \end{array}$$



Imperative

First-order recursive filter



Block diagram representation

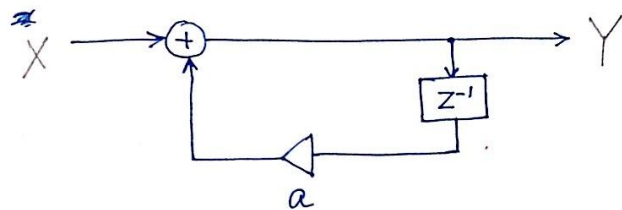
Difference equation:

$$y[n] = a \cdot y[n-1] + x[n]$$

The current value of the output is equal to the sum of the input and 'a' times the value of the previous output. As the current output is also a function of the previous outputs, it is called a recursive filter.

The term order is used to denote the minimum number of 'delays' and 'advances' that are necessary to implement a digital filter. Since, only one delay is necessary (here to implement), this is a first order filter.

First-order Recursive Filter



Operator approach:

$$Y = X + aDY$$

$$\Rightarrow Y - aDY = X$$

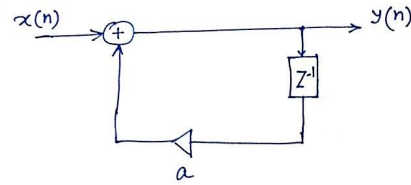
$$\Rightarrow (1 - aD)Y = X$$

$$\Rightarrow \frac{Y}{X} = \frac{1}{1 - aD} \quad \leftarrow \text{Compare it with } H(z) = \frac{1}{1 - az^{-1}}$$

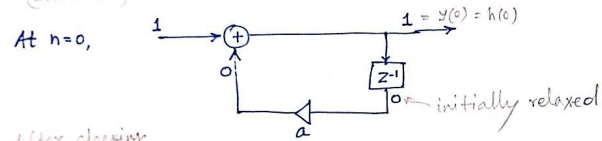
System functional

$$D \equiv Z^{-1}$$

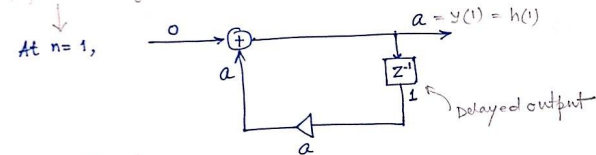
First-order Recursive Filter :



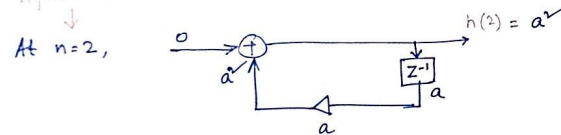
Informal Approach : (Sample by sample basis) (Intuitive)



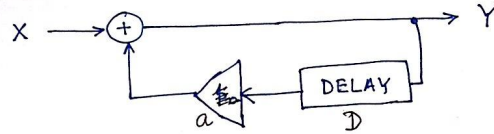
After clocking



After another clock



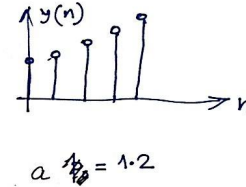
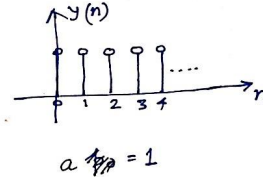
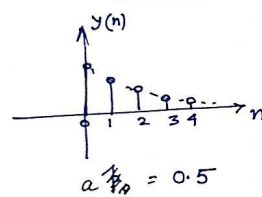
Hence,
$$h(n) = a^n u(n)$$



ii) $h(n) = a^n$, if $n \geq 0$;
 $= 0$, otherwise

$\frac{1}{1-aD}$ is called the pole

or, $h(n) = a^n u(n)$, $\forall n$

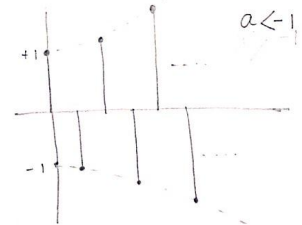
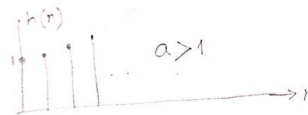
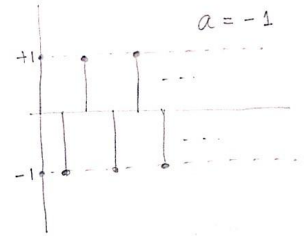
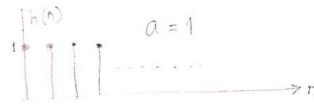
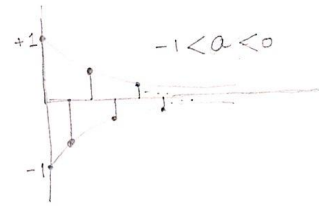


using operator notation we can write the following —

i) $X + aDY = Y$
 $\Rightarrow X = (1 - aD)Y$

$\Rightarrow \frac{Y}{X} = \frac{1}{1 - aD}$ for 1st order filter

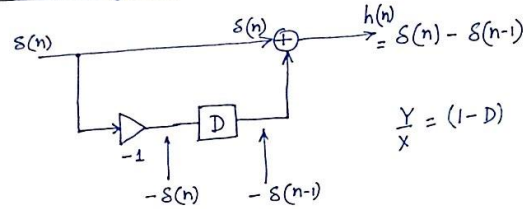
Sketch of $h(n) = a^n u(n)$: $-1 \quad 0 \quad 1$



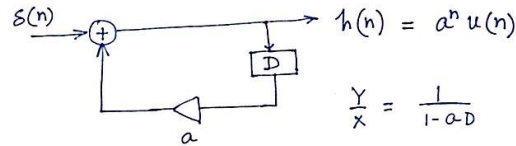
Note: To have a stable filter "a" must be numerically less than '1', sign may be +ve or -ve.

To obtain impulse (unit sample) response of a filter from its block diagram by inspection

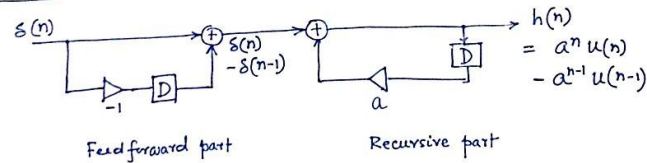
i) Non-recursive filter :



ii) Recursive first-order filter :



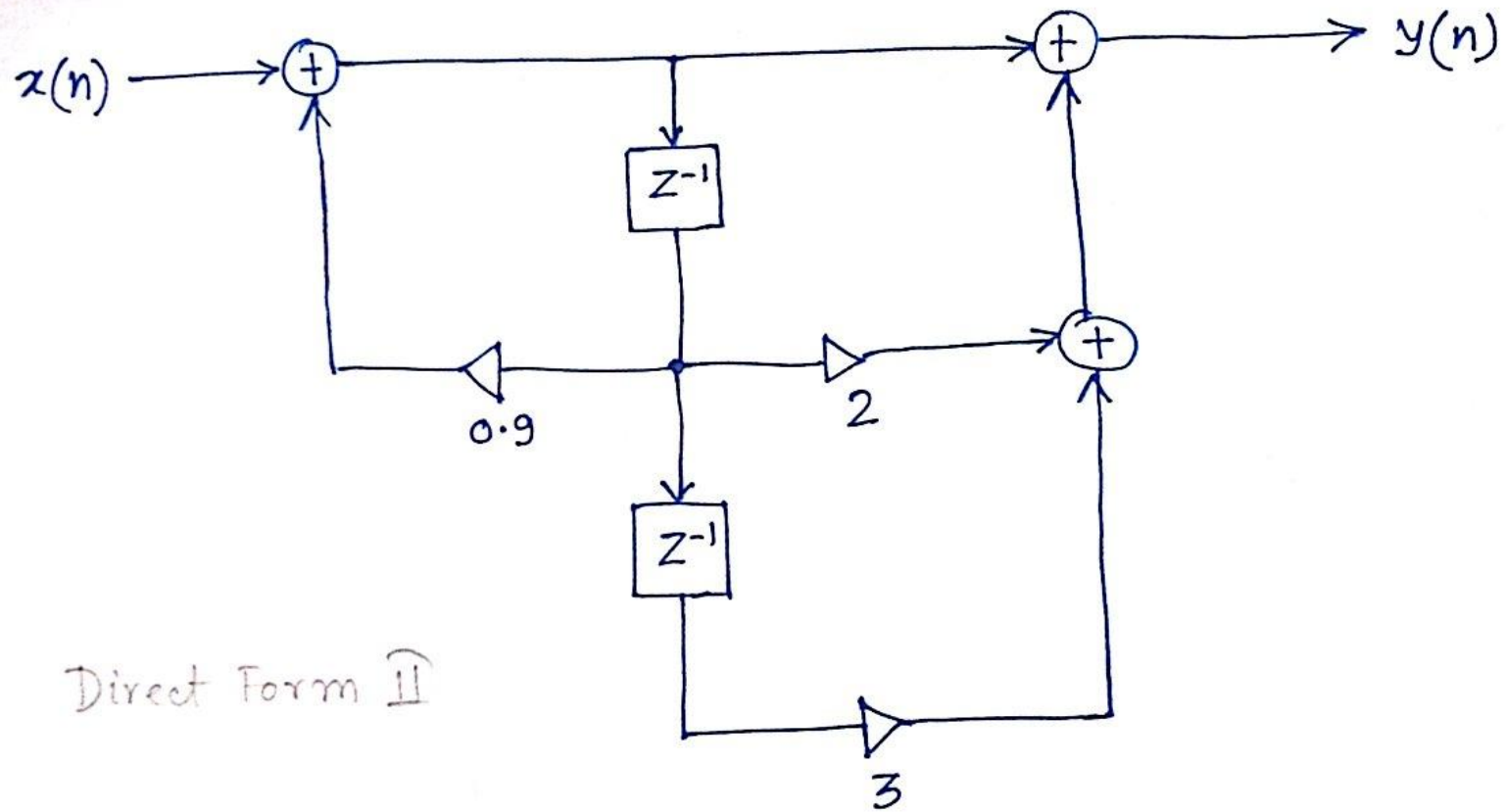
iii) ^(Combined)
General first-order filter :



Filter (realization) structures

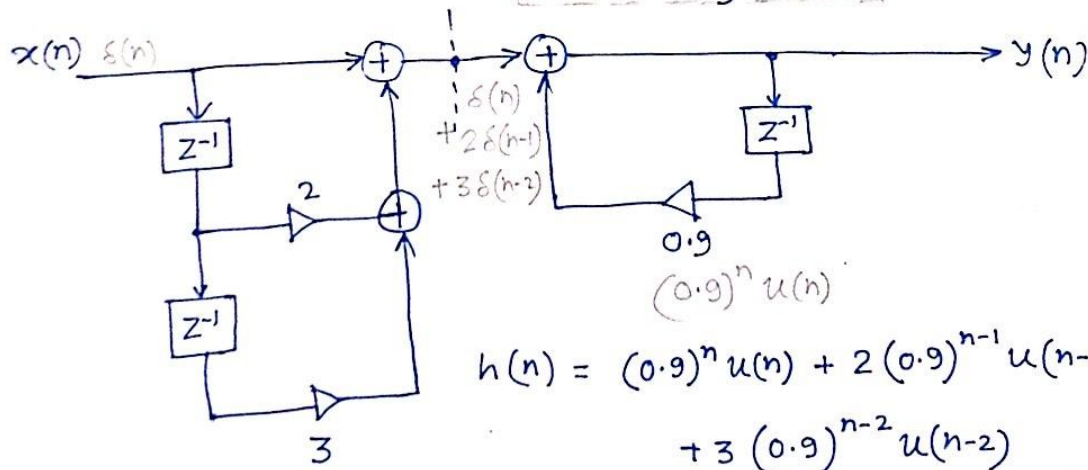
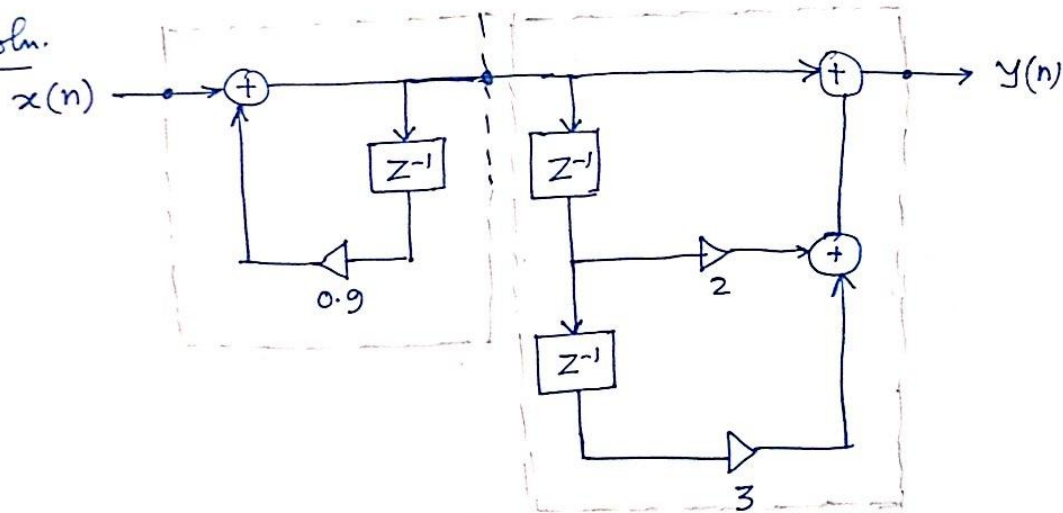
- Direct form I
- Direct form II
- Series (cascade) form
- Parallel form

Prob. Obtain the impulse response of the following filter.



Direct Form II

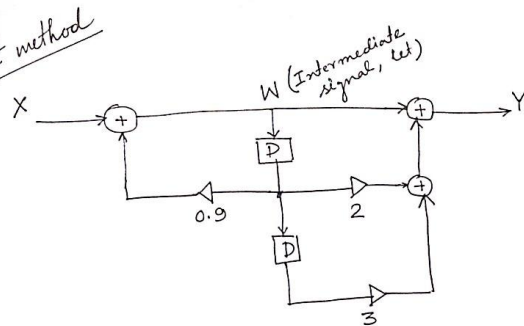
Soln.



$$h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$$

Direct Form I

Alt method



Using operator notation,

$$\begin{aligned} 0.9DW + X &= W \\ \Rightarrow X &= (1 - 0.9D)W \quad \text{--- (i)} \end{aligned}$$

$$\begin{aligned} W + 2DW + 3D^2W &= Y \\ \Rightarrow (1 + 2D + 3D^2)W &= Y \quad \text{--- (ii)} \end{aligned}$$

$$Y = (1 + 2D + 3D^2)W = (1 + 2D + 3D^2) \times \frac{X}{1 - 0.9D}$$

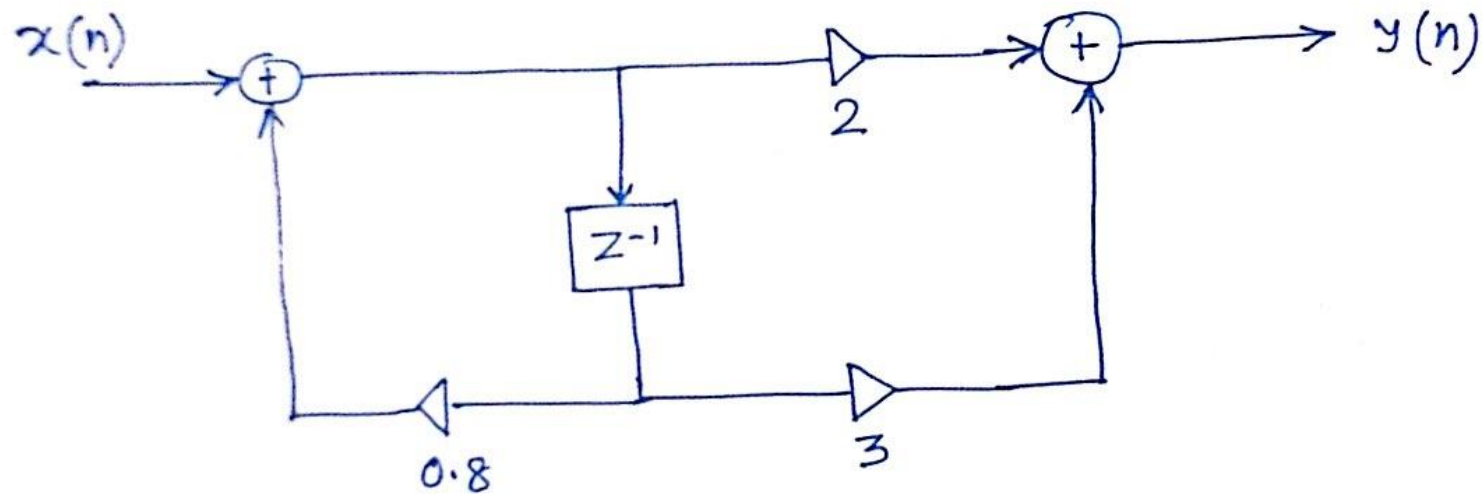
$$\Rightarrow \frac{Y}{X} = \frac{1 + 2D + 3D^2}{1 - 0.9D} \quad \Leftarrow \text{System functional}$$

$$\Rightarrow (1 - 0.9D)Y = (1 + 2D + 3D^2)X$$

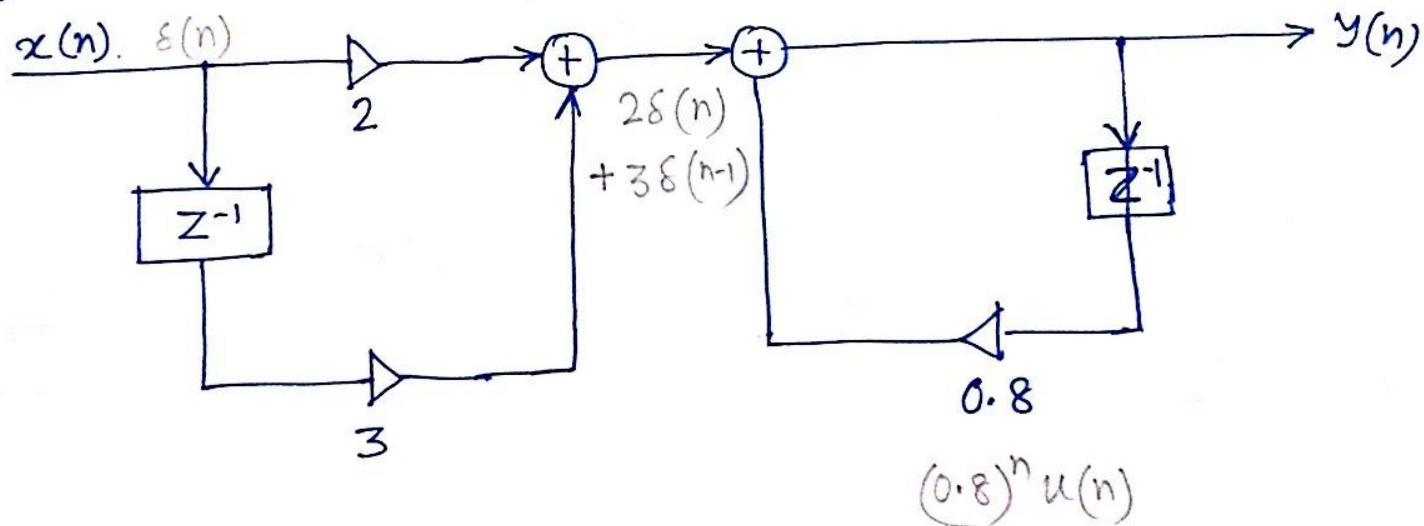
$$\Rightarrow y(n) - 0.9y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$$

$$\Rightarrow y(n) = x(n) + 2x(n-1) + 3x(n-2) + 0.9y(n-1)$$

Prob. Obtain the impulse response of the following filter.



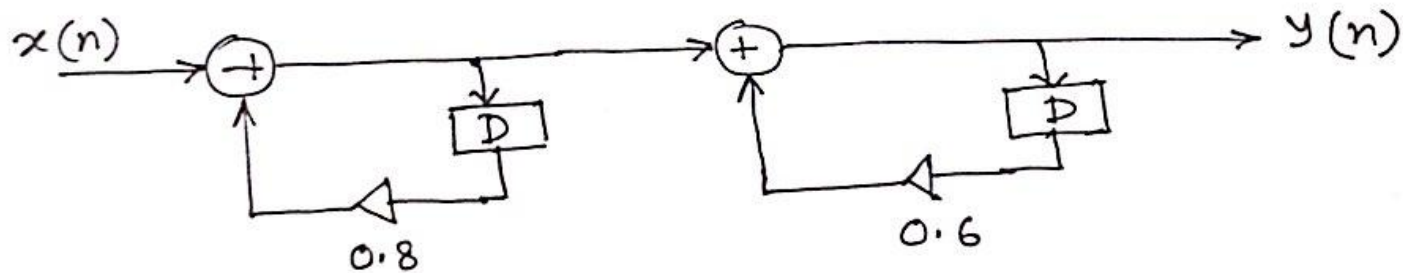
Soln.



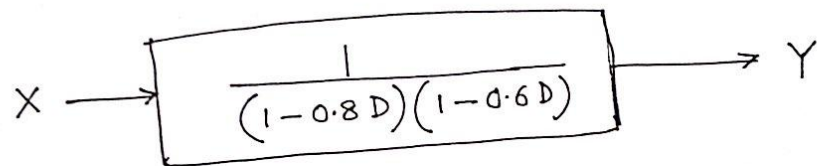
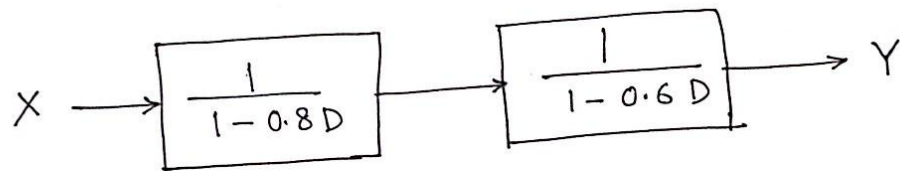
$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$

Problem: Block diagram of a DT system is given below.

- Determine the difference equation.
- Obtain the impulse response.



Soln.
a)



$$\frac{Y}{X} = \frac{1}{(1-0.8D)(1-0.6D)}$$

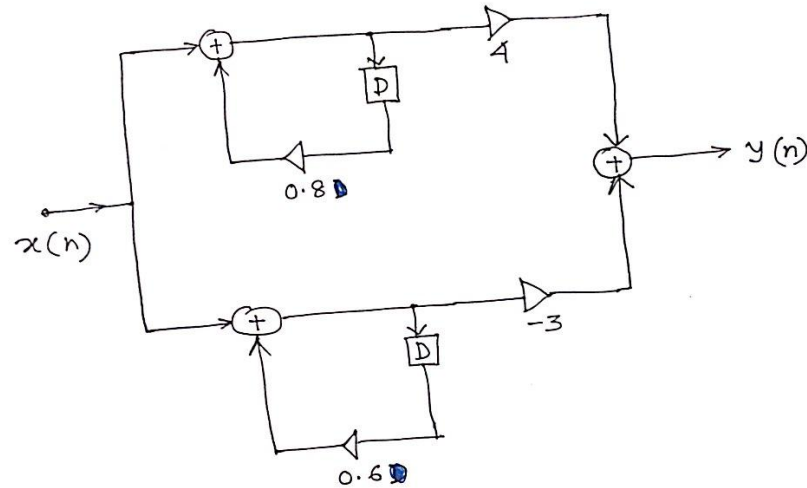
$$\Rightarrow (1-0.8D)(1-0.6D) Y = X$$

$$\Rightarrow (1 - 1.4D + 0.48D^2) Y = X$$

$$\Rightarrow y(n) - 1.4 y(n-1) + 0.48 y(n-2) = x(n)$$

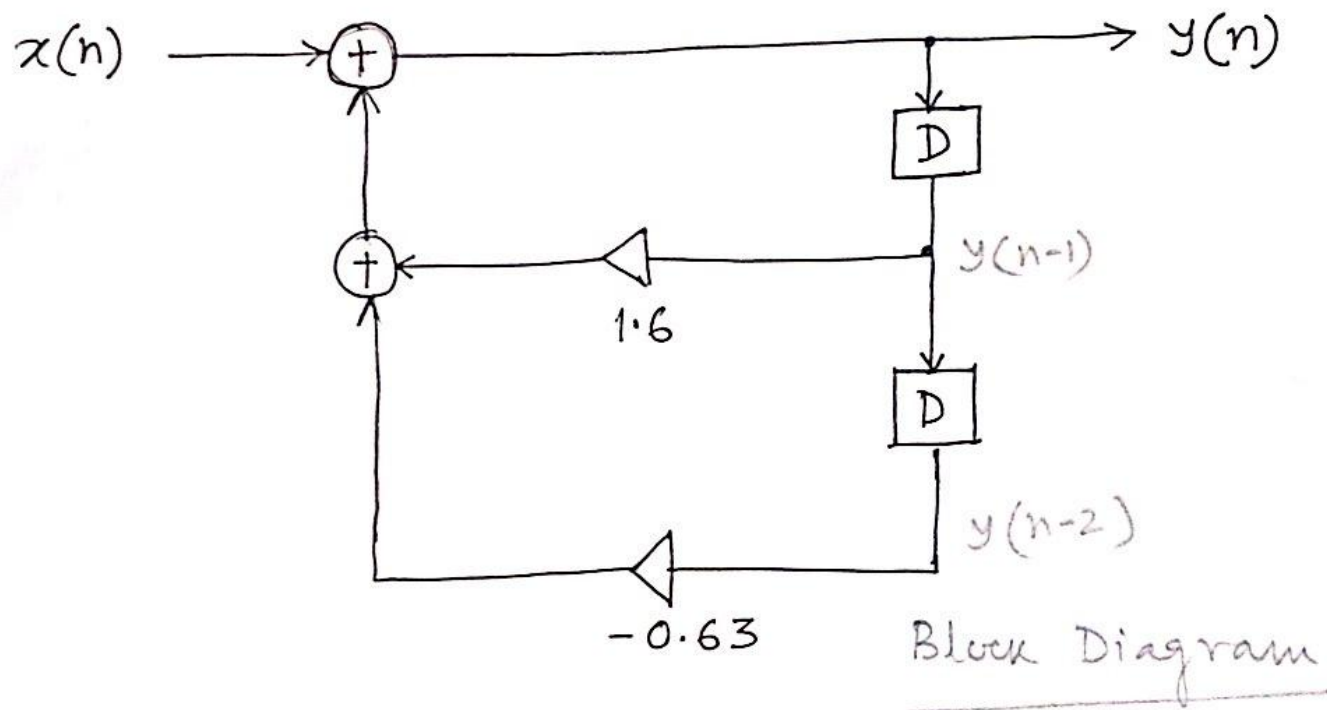
$$\Rightarrow y(n) = x(n) + 1.4 y(n-1) - 0.48 y(n-2)$$

$$b) \quad \frac{Y}{X} = \frac{1}{(1-0.8D)(1-0.6D)} = \frac{4}{1-0.8D} + \frac{-3}{1-0.6D}$$

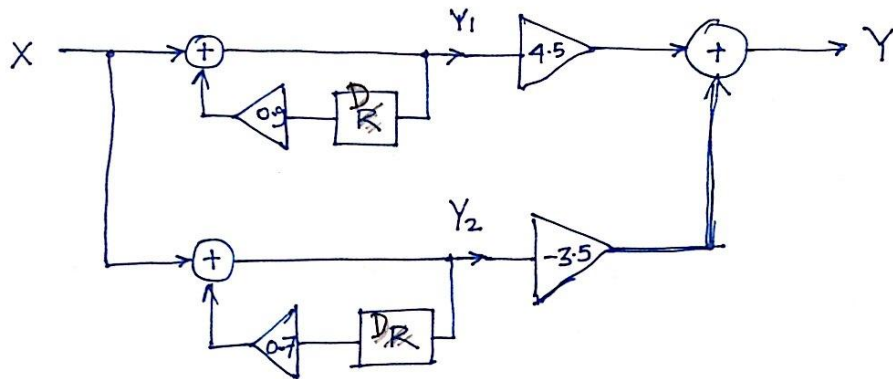


$$h[n] = [4(0.8)^n - 3(0.6)^n] u[n] \quad \forall n$$

Second Order System



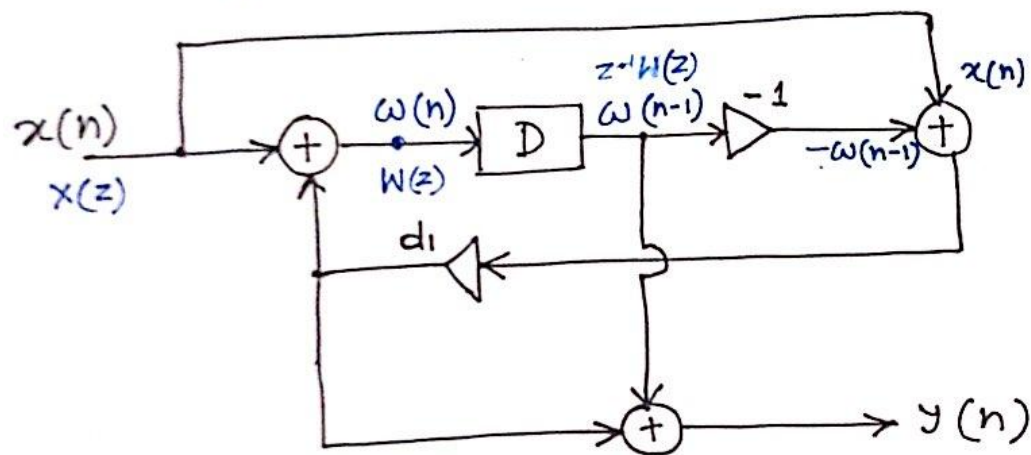
$$\begin{aligned}\frac{Y}{X} &= \frac{1}{(1-0.9D)(1-0.7D)} = \frac{4.5}{1-0.9D} - \frac{3.5}{1-0.7D} \\ &= 4.5 \times \frac{1}{1-0.9D} + (-3.5) \times \frac{1}{1-0.7D}\end{aligned}$$



If $x(n) = \delta(n)$ then $y_1(n) = 0.9^n$
and $y_2(n) = 0.7^n$ for $n \geq 0$

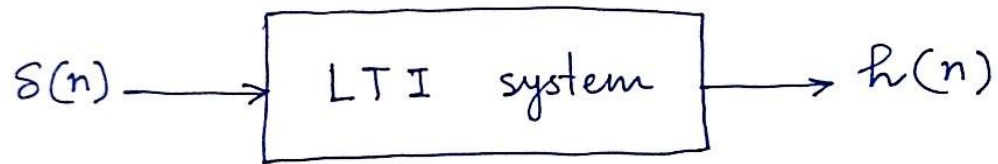
$$\begin{aligned}\text{Thus, } y(n) &= 4.5y_1(n) - 3.5y_2(n) \\ &= 4.5(0.9)^n - 3.5(0.7)^n \quad \text{for } n \geq 0\end{aligned}$$

Problem: Develop the relation between $y(n]$ and $x(n]$ from the following block diagram of a discrete-time system.



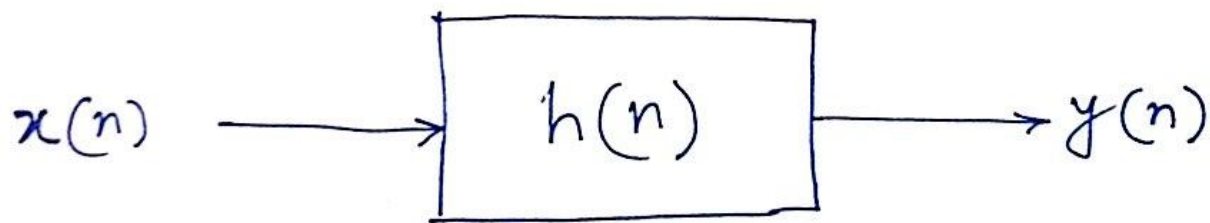
System representation using its impulse response:

A linear time-invariant (LTI) system can be completely described by its unit-impulse response, which is defined ~~by~~ as the system response due to the impulse input $\delta(n)$ with zero initial conditions.



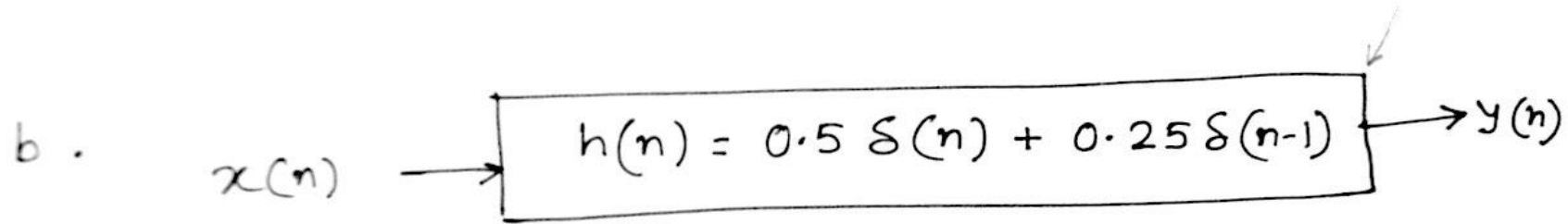
unit-impulse response

With the obtained unit-impulse response $h(n)$, we can represent the linear time-invariant system.

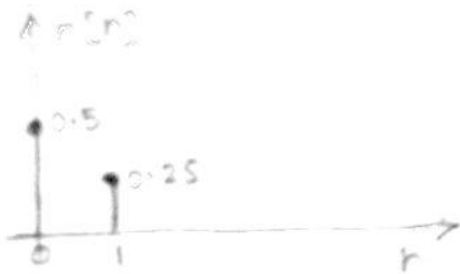


system representation

FIR System (FIR Filter)



• It contains finite number of terms and hence
it is called a finite impulse response (FIR) system



$$\Rightarrow h[n] = 0.5 \delta[n] + 0.25 \delta[n-1]$$

IIR System (IIR Filter)

$$\begin{aligned}h(n) &= (0.25)^n u(n) \\&= \delta(n) + 0.25 \delta(n-1) + (0.25)^2 \delta(n-2) + \dots\end{aligned}$$

expressed in $\delta(n)$

