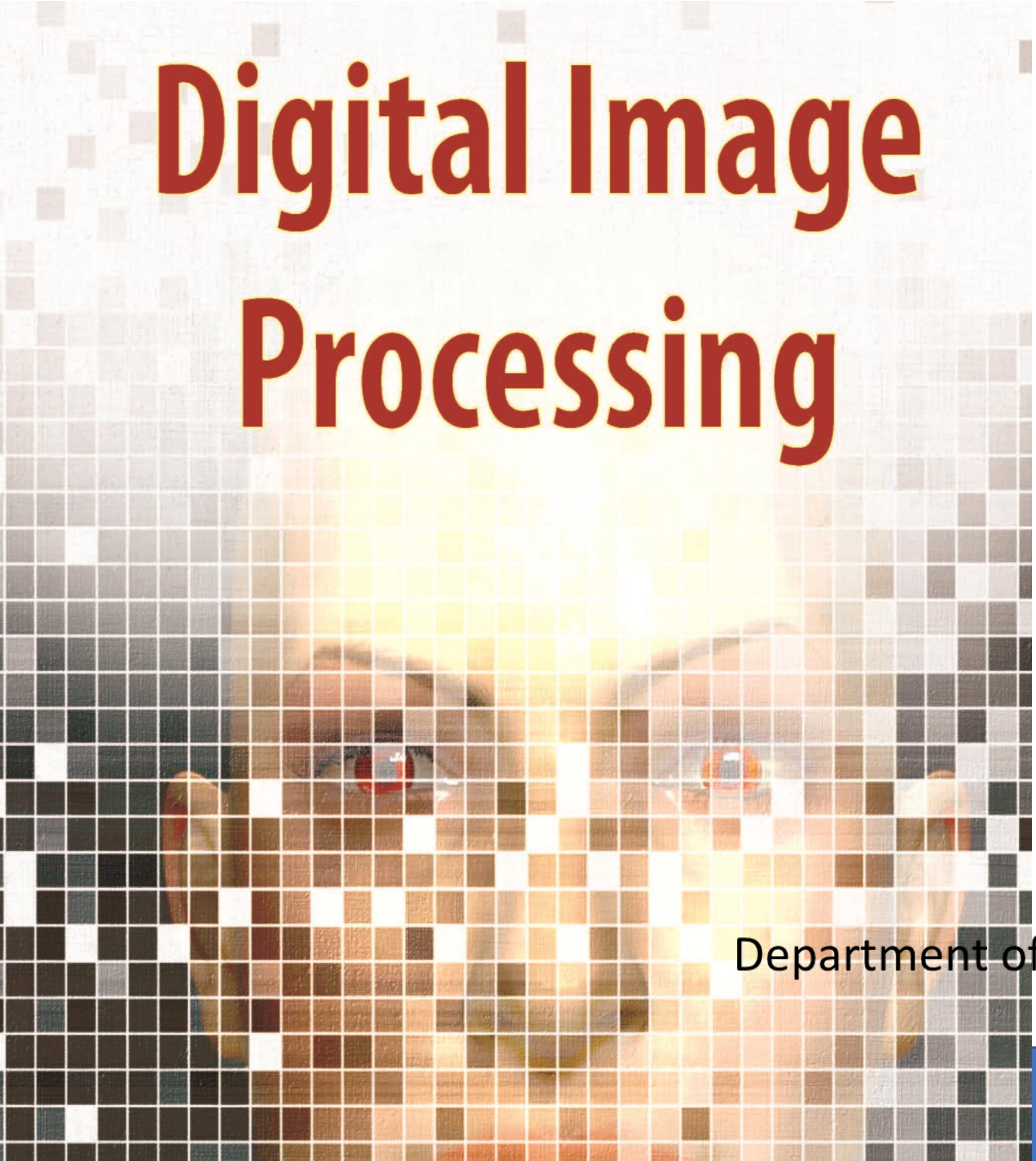


Intensity Transforms

Prof. Sheli Sinha Chaudhuri
Department of Electronics and Telecommunication Engineering
Jadavpur University

Chapter 3

Lecture 4: Intensity Transforms

Contents

- Basic Intensity Transformation Functions
- Histogram Processing
- Fundamentals of Spatial Filtering
- Combined Spatial Enhancement Methods

Image Enhancement (Spatial)

- **Image enhancement:**

1. Improving the interpretability or perception of information in images for human viewers
2. Providing `better' input for other automated image processing techniques

- **Spatial domain methods**:

  Operate directly on pixels

- **Frequency domain methods**:

  Operate on the Fourier transform of an image

Point Processing

- The simplest kind of range transformations are those independent of position *x,y*:

$$g = T(f)$$

- This is called point processing.

- **Important:** every pixel for itself – spatial information completely lost!

Spatial Operations

- Single-pixel operation (Intensity Transformation)
  - Negative Image, contrast stretching etc.
- Neighborhood operations
  - Averaging filter, median filtering etc.
- Geometric spatial transformations
  - Scaling, Rotation, Translations etc

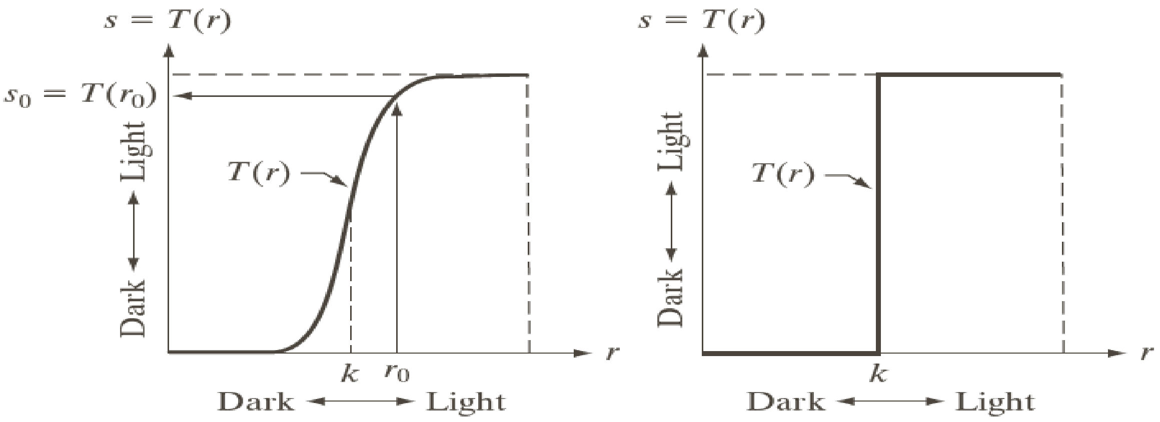
Point Processing

- Spatial Domain methods operate directly on image pixels

$$G(x,y)=T[f(x,y)]$$

- T operates only on one pixel

$$S=T(r)$$

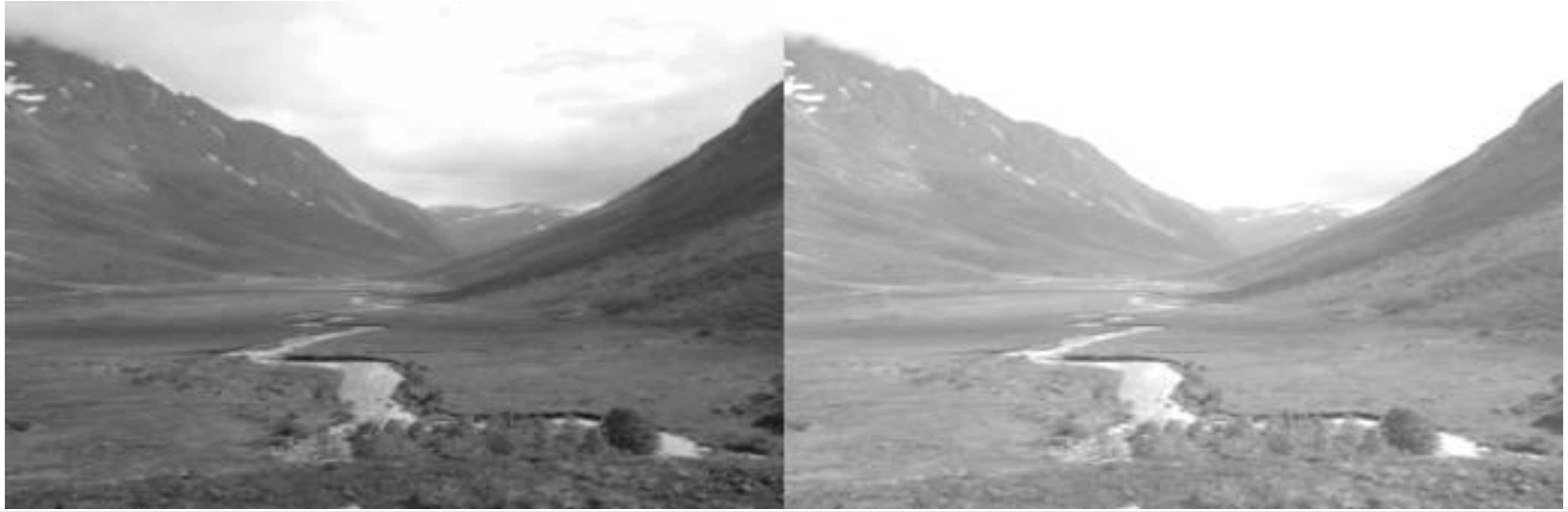


a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

Single Pixel Operations

When changing the brightness of an image, a constant is added or subtracted from the luminance of all sample values. This is equivalent to shifting the contents of the histogram left (subtraction) or right (addition).



Neighborhood Operations

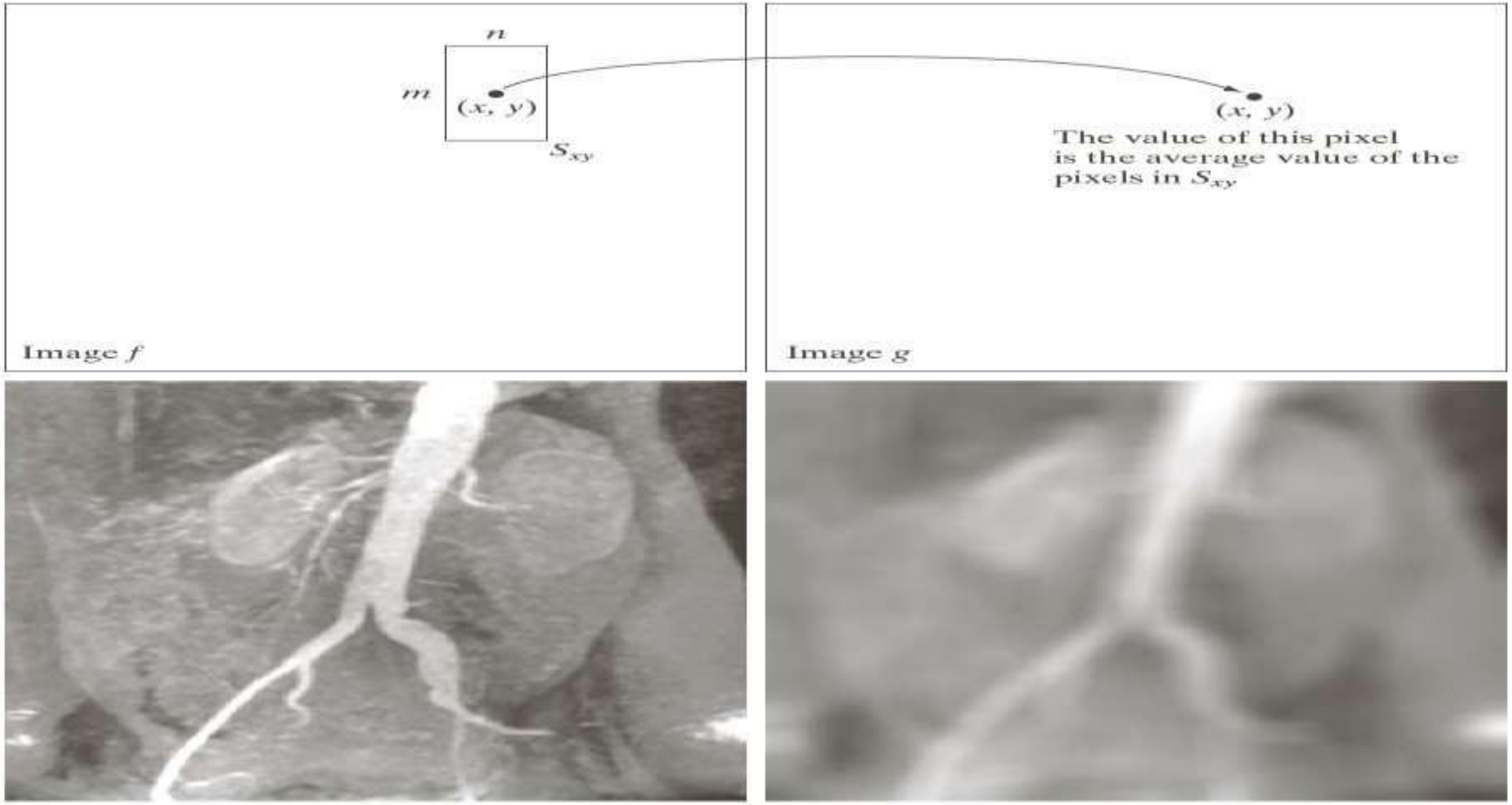


Image Enhancement

- The objective of image enhancement is to process an image so that the result is more *suitable* than the original image for a specific application.
- There are two main approaches:
  - Image enhancement in spatial domain: Direct manipulation of pixels in an image
    - Point processing: Change pixel intensities
    - Spatial filtering
  - Image enhancement in frequency domain: Modifying the Fourier transform of an image

Some Basic Intensity Transformation Functions

- Image Negatives

  $s = L - 1 - r$

  - S is the output intensity value
  - L is the highest intensity levels
  - r is the input intensity value
  - Particularly suited for enhancing white or gray detail embedded in dark regions of an image, especially when the black areas are dominant in size

Some Basic Intensity Transformation Functions

- Image Negatives

Some Basic Intensity Transformation Functions

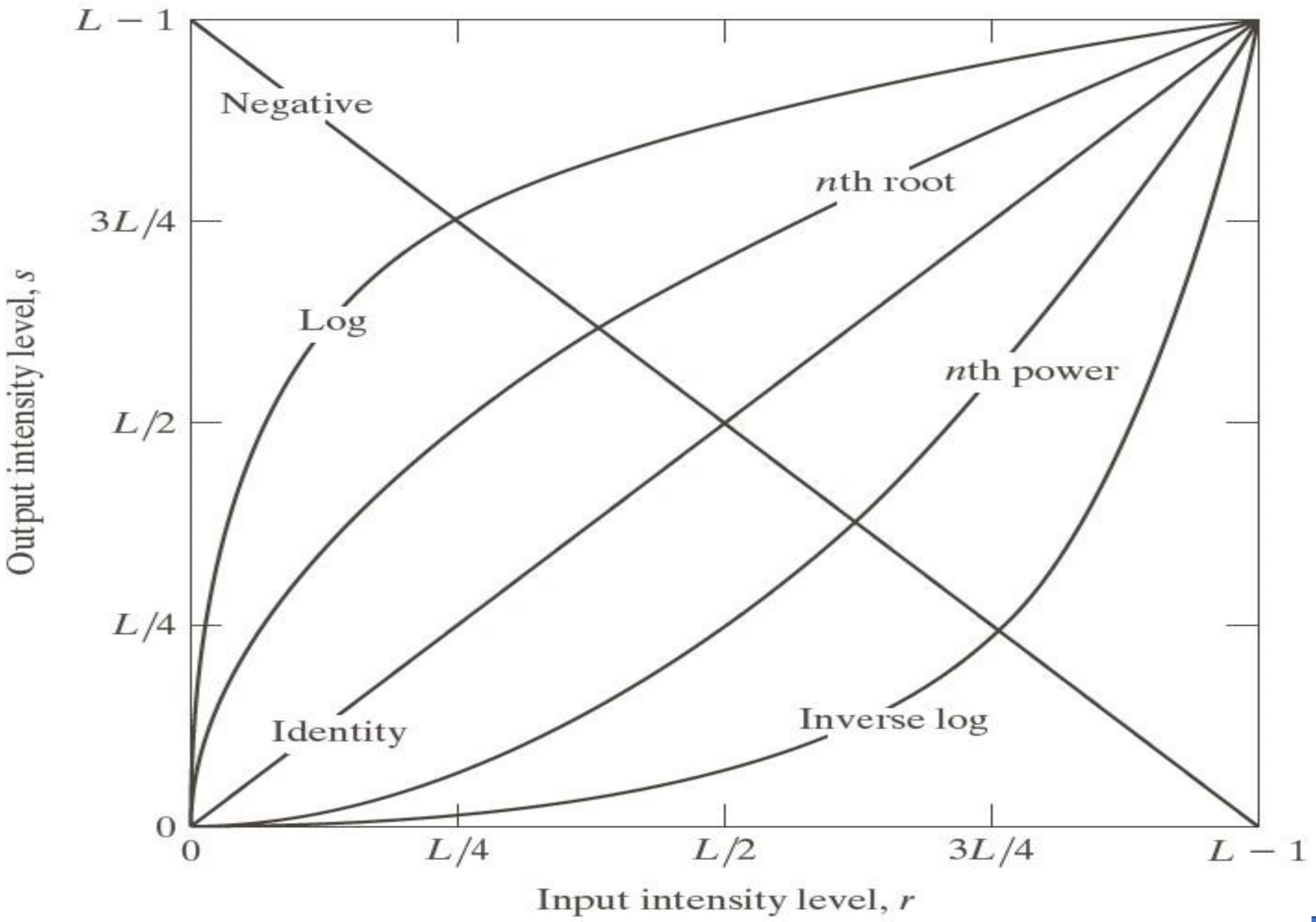


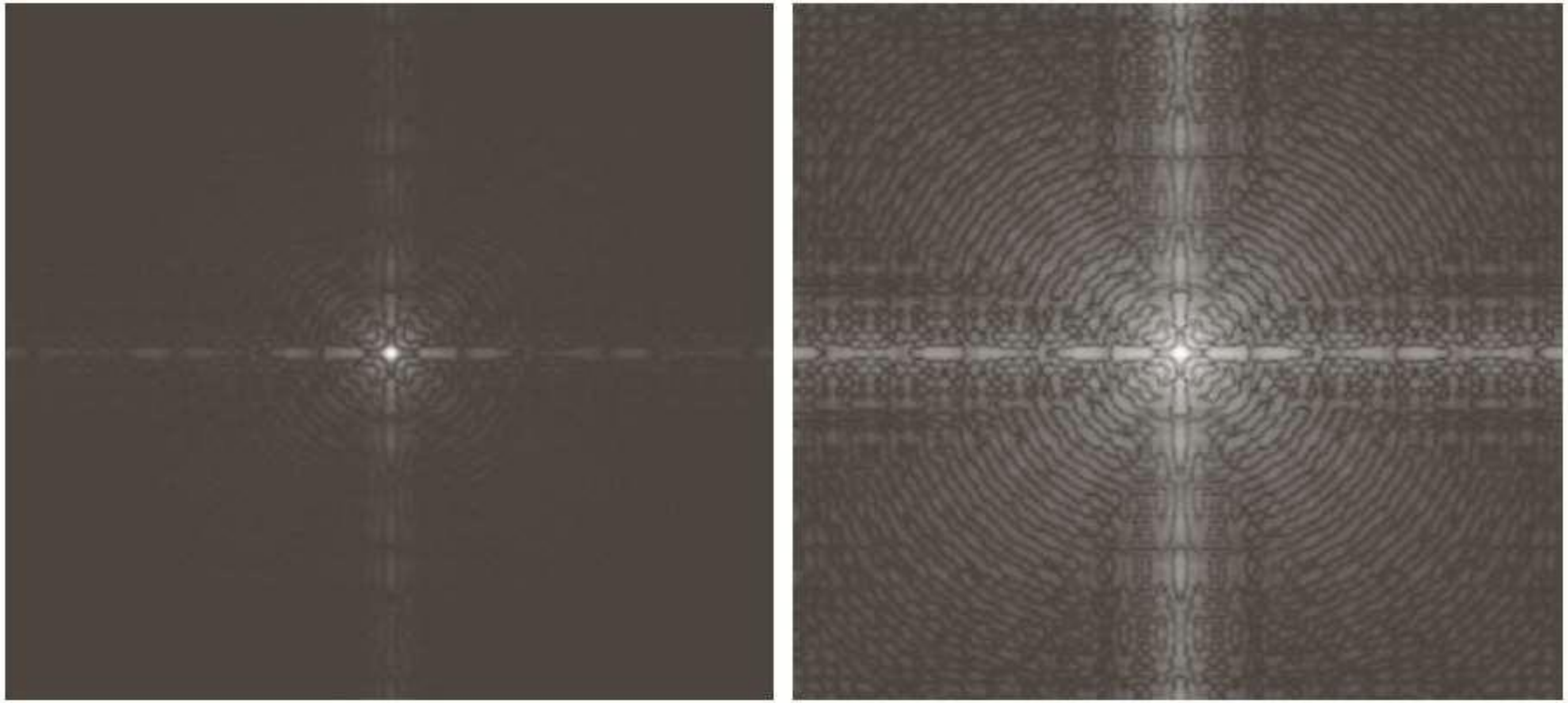
FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Some Basic Intensity Transformation Functions

- Log Transformations
  - $s = c \log(1 + r)$
    - c is constant
  - It maps a narrow range of low intensity values in the input into a wide range of output levels
  - The opposite is true of higher values of input levels
  - It expands the values of dark pixels in an image while compressing the higher level values
  - It compresses the dynamic range of images with large variations in pixel values

Some Basic Intensity Transformation Functions

- Log Transformations



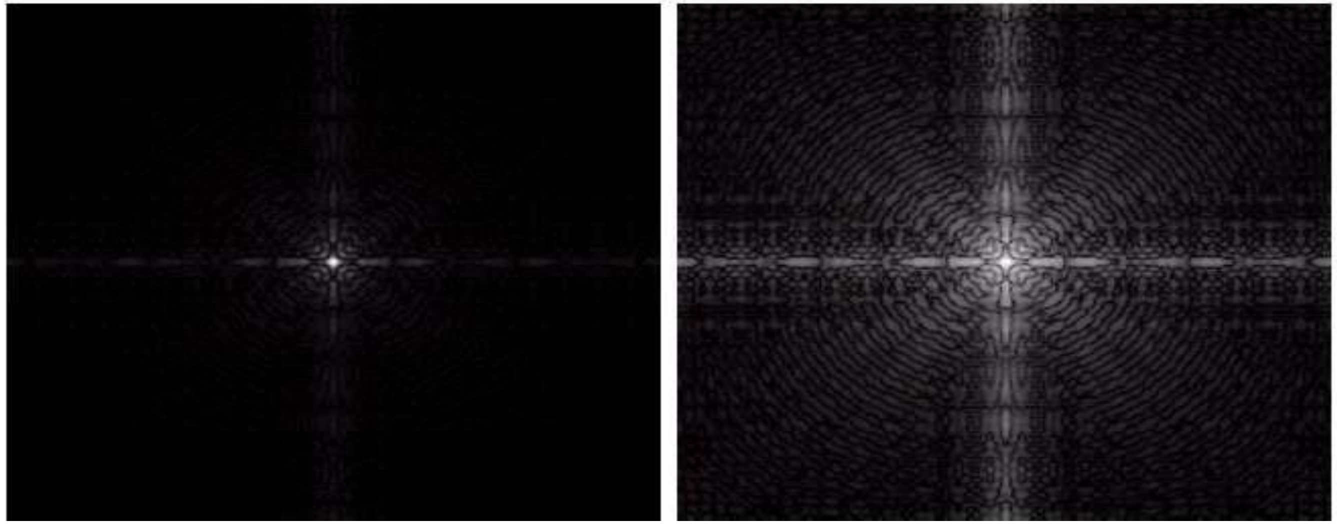
a b

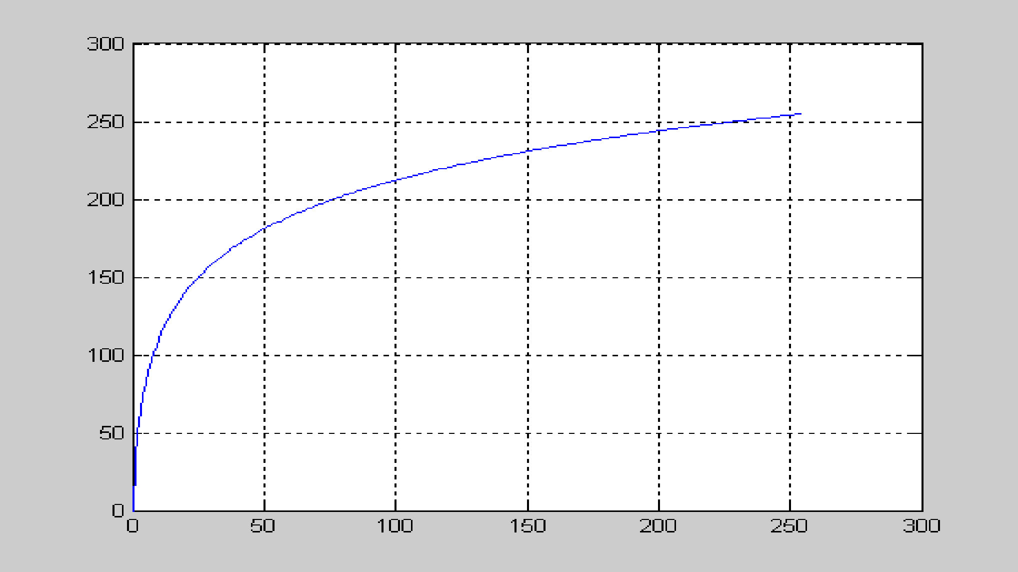
FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

Log Transform

a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.





$$T(r) = c\log(1+r)$$

Some Basic Intensity Transformation Functions

- Power Law (Gamma) Transformations
  - $s = c\, r^{\gamma}$
    - c and γ are both positive constants
  - With fractional values($0<\gamma<1$) of gamma map a narrow range of dark input values into a wider range of output values, with the opposite being true for higher values ($\gamma > 1$)of input levels.
  - C=gamma=1 means it is an identity transformations.
  - Variety of devices used for image capture , printing, and display respond according to a power law.
  - Process used to correct these power law response phenomena is called *gamma correction.*

Some Basic Intensity Transformation Functions

- Power Law (Gamma) Transformations

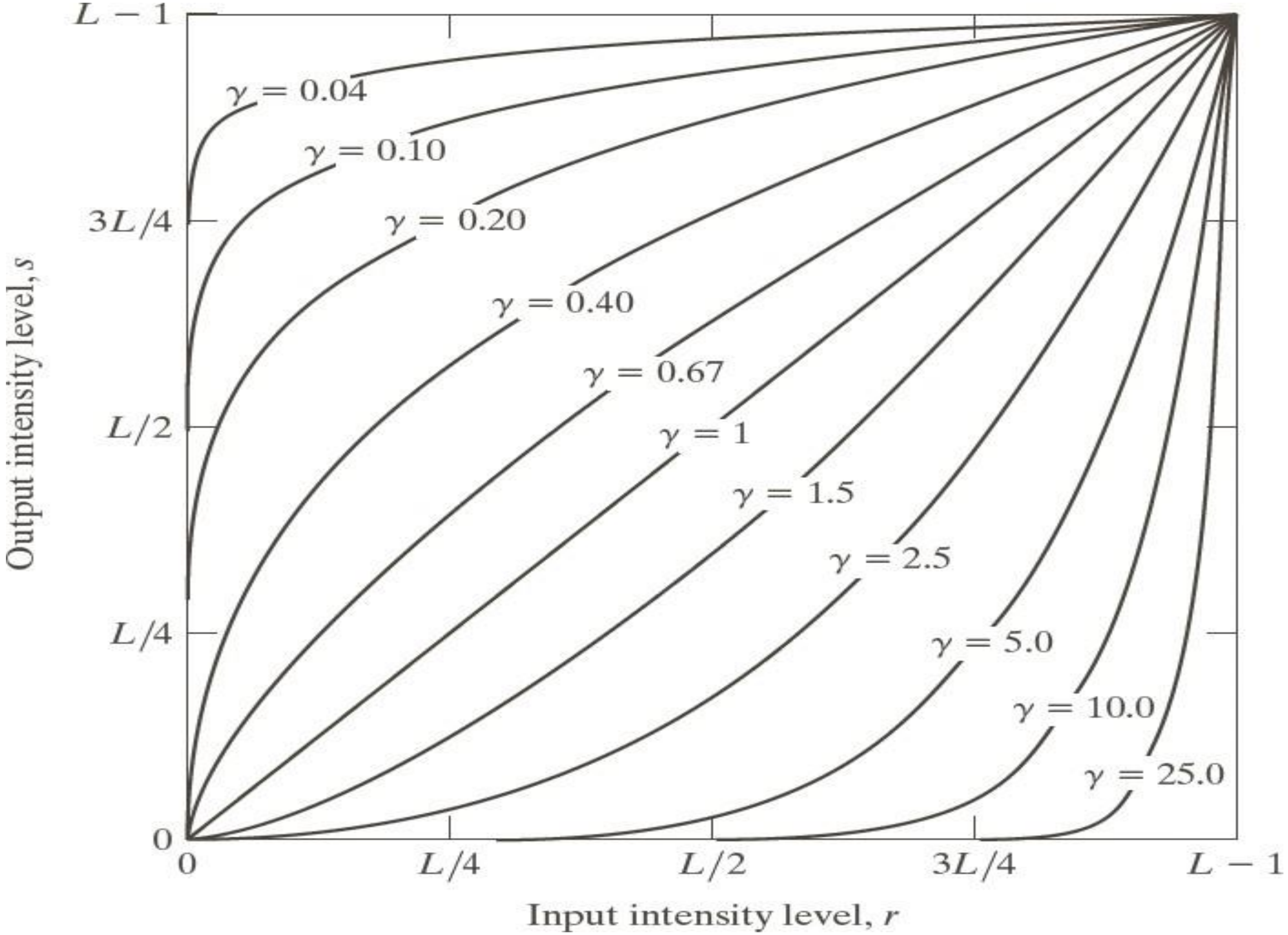
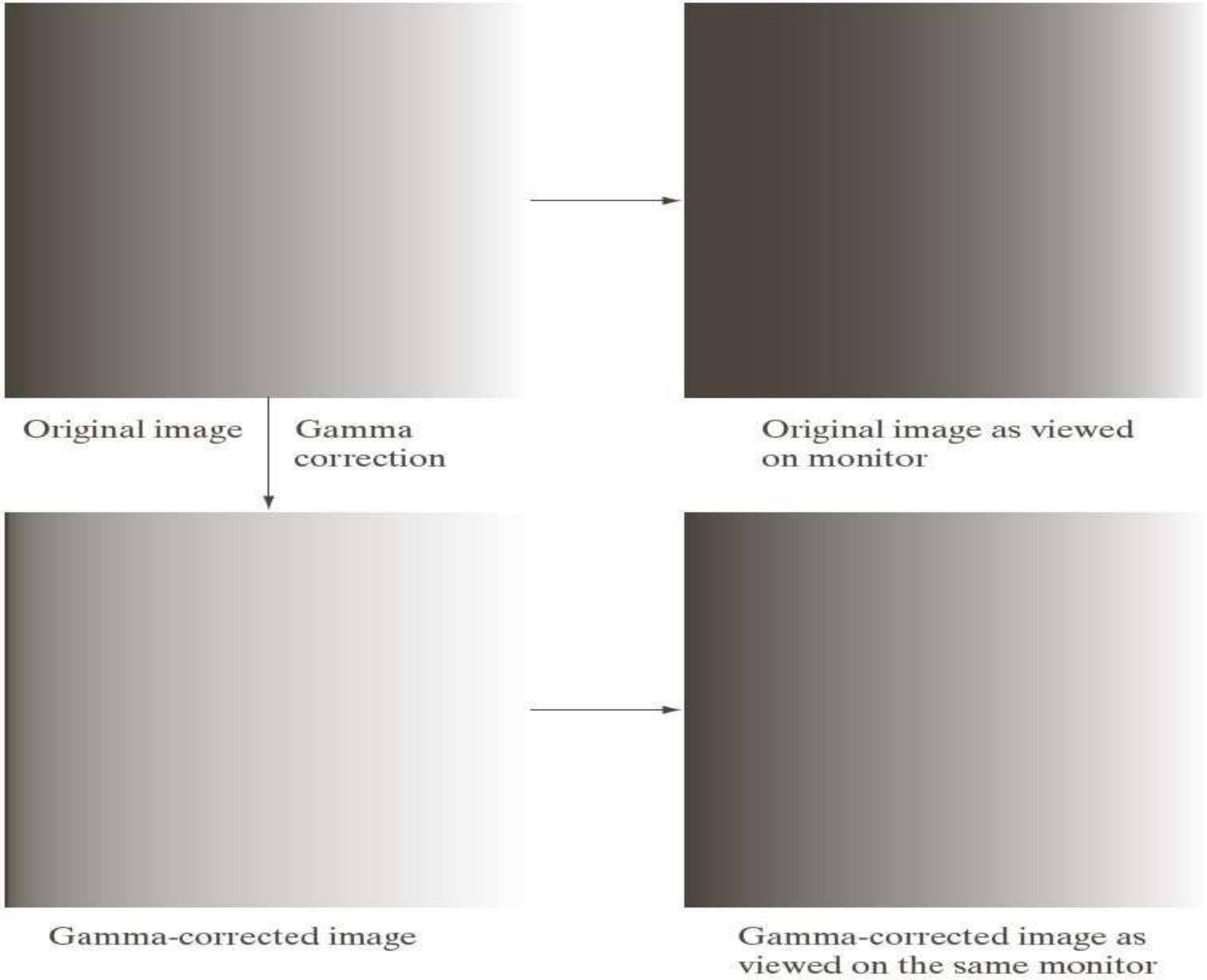


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Some Basic Intensity Transformation Functions



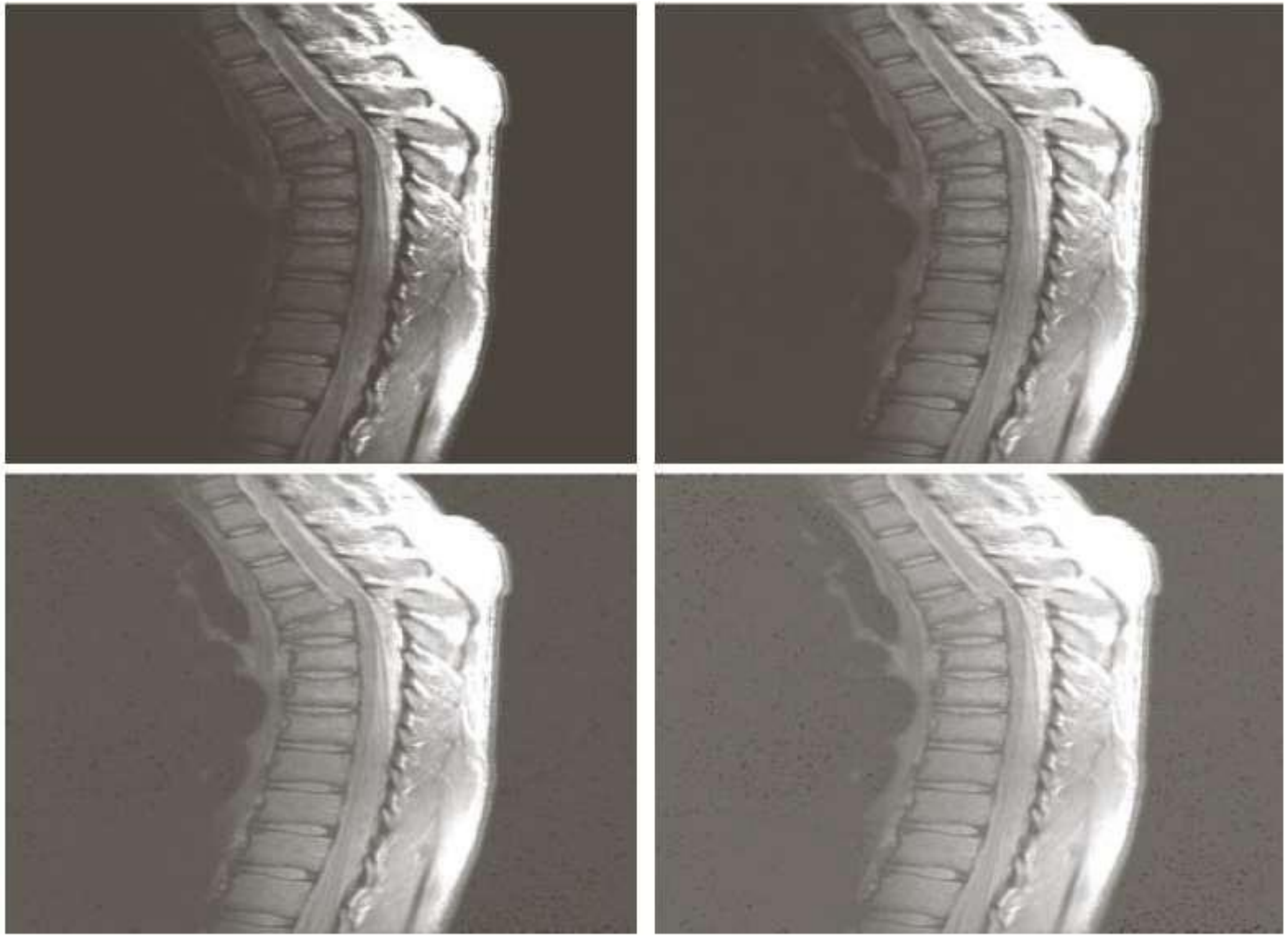
a	b
c	d

FIGURE 3.7 (a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

Some Basic Intensity Transformation Functions

- Power Law (Gamma) Transformations
  - Images that are not corrected properly look either bleached out or too dark.
  - Varying gamma changes not only intensity, but also the ratio of red to green to blue in a color images.
  - Gamma correction has become increasingly important, as the use of the digital images over internet.
  - Useful for general purpose contrast manipulation.
  - Apply gamma correction on CRT (Television, monitor), printers, scanners etc.
  - Gamma value depends on device.

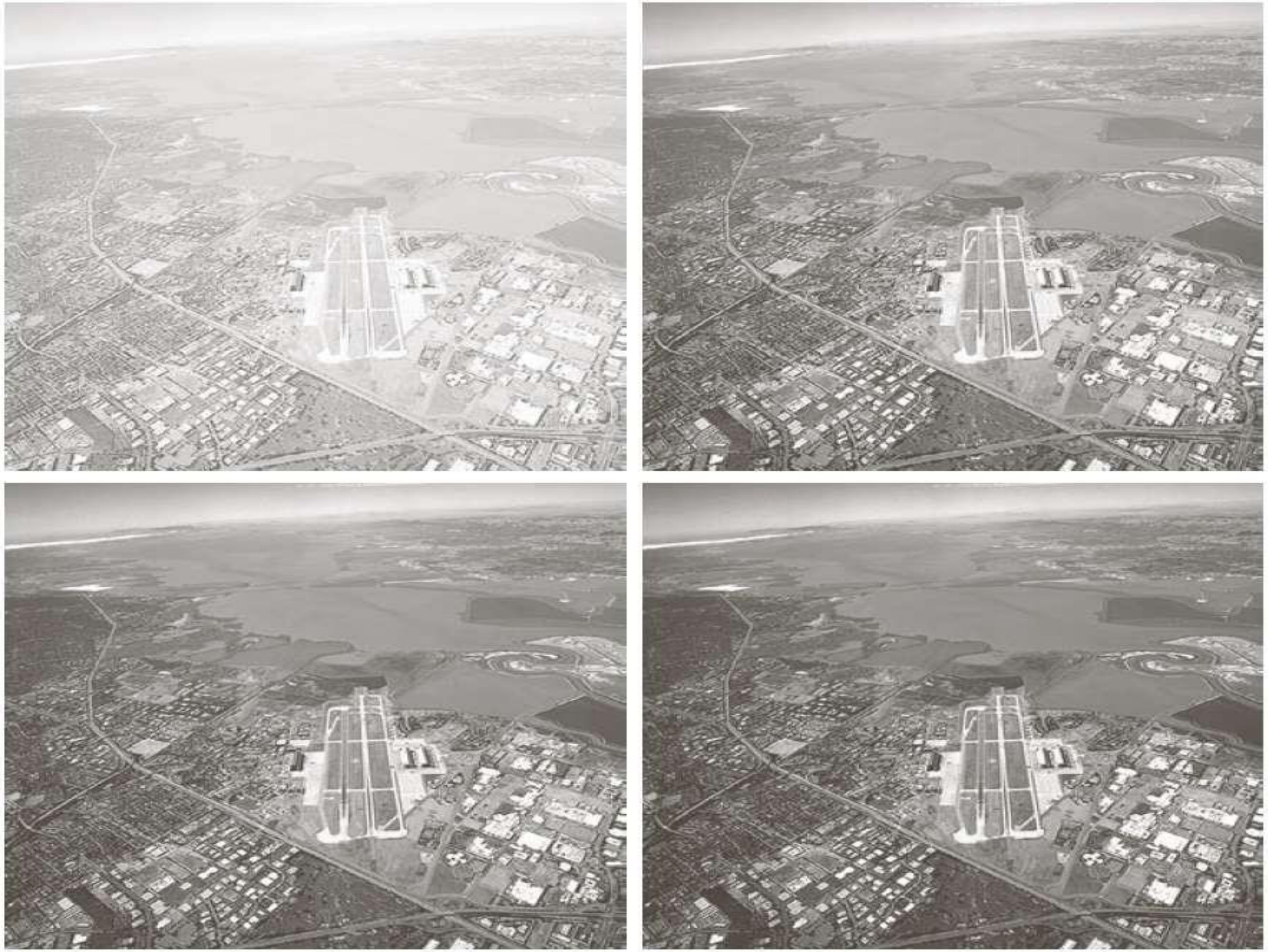
Some Basic Intensity Transformation Functions



a b
c d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Some Basic Intensity Transformation Functions



a	b
c	d

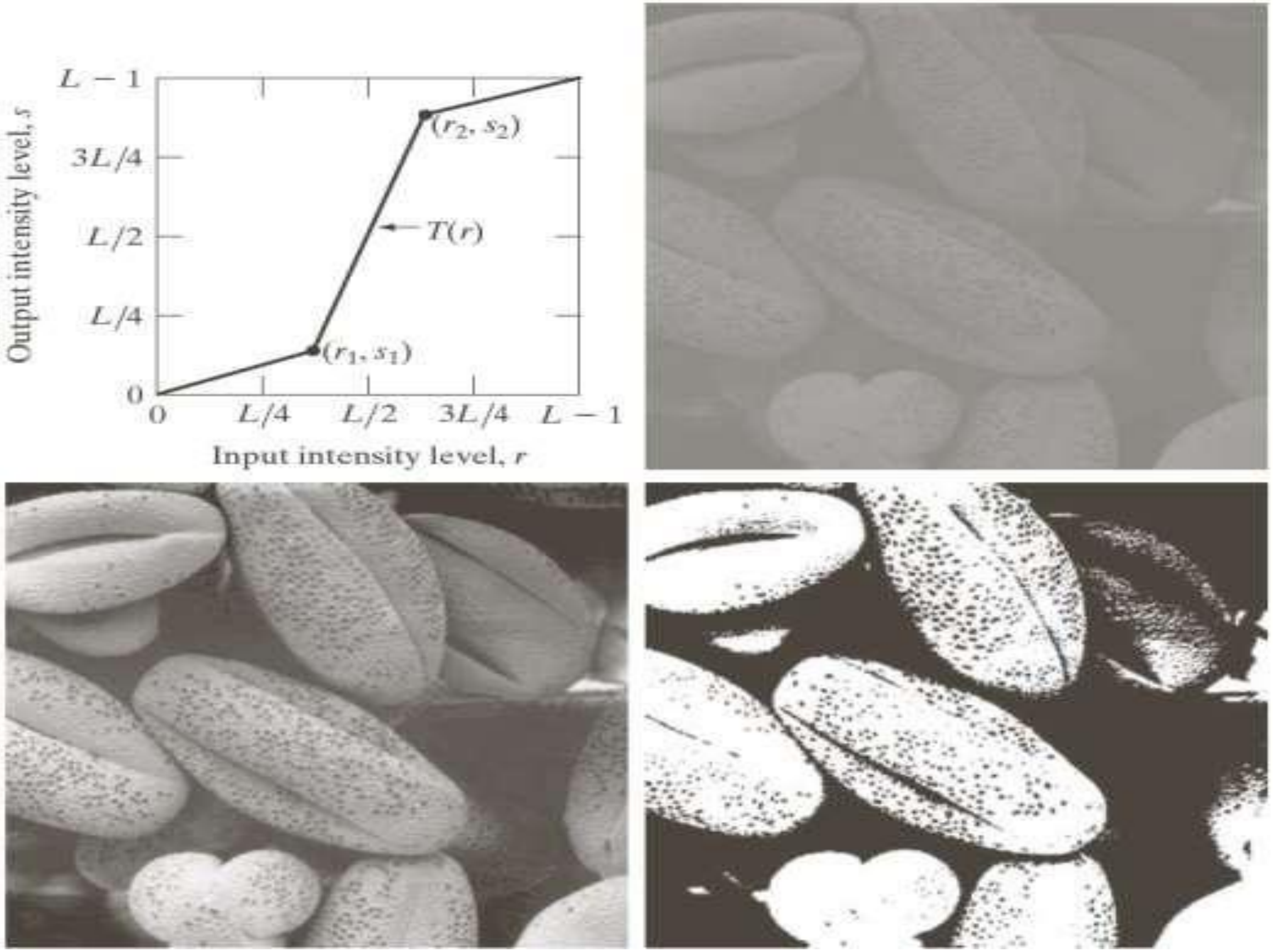
FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Piecewise-Linear Transformation Functions

- Contrast Stretching
  - Low contrast images can result from poor illuminations.
  - Lack of dynamic range in the imaging sensor, or even the wrong setting of a lens aperture during image acquisition.
  - It expands the range of intensity levels in an image so that it spans the full intensity range of display devices.
  - Contrast stretching is obtained by setting

$$(r1,s1) = (r_{min}, 0) \text{ and } (r2,s2) = (r_{max}, L-1)$$

Piecewise-Linear Transformation Functions



a	b
c	d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Piecewise-Linear Transformation Functions

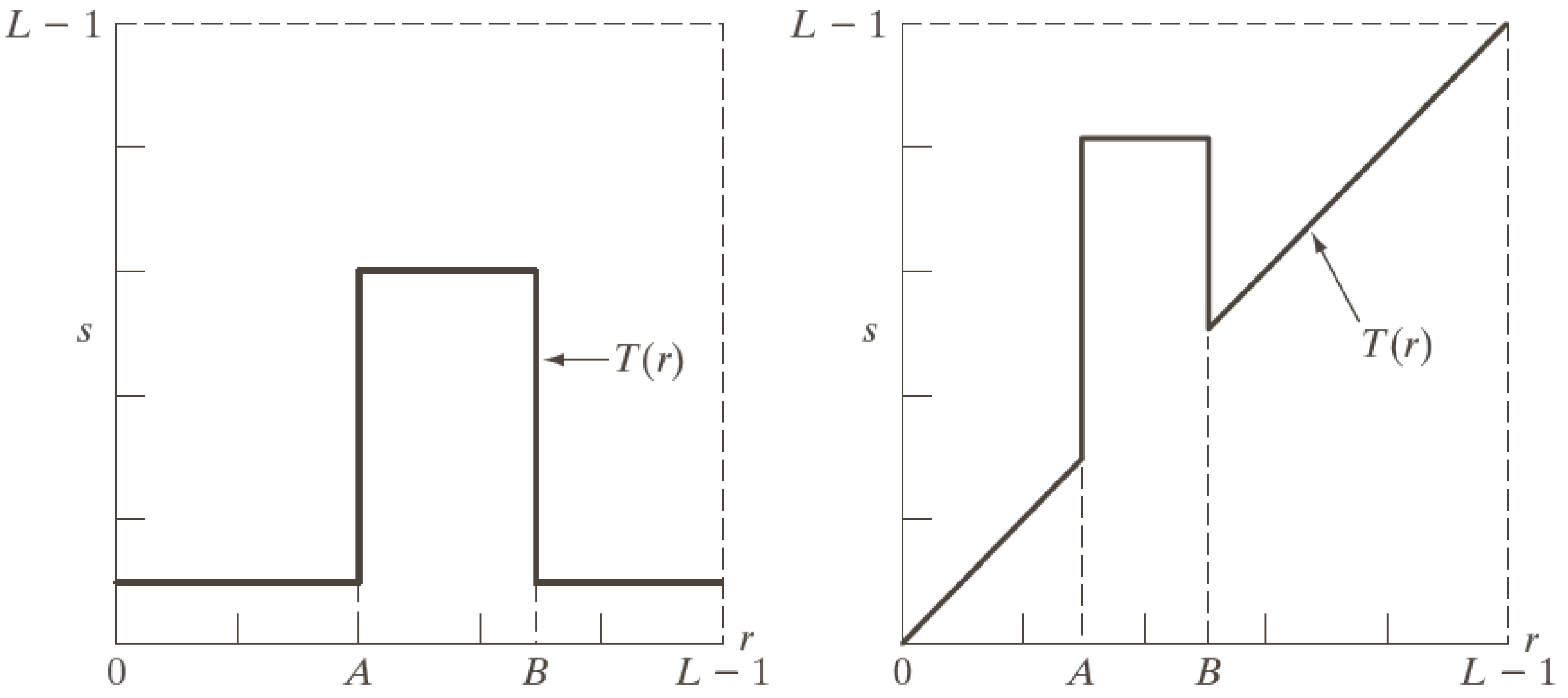
- Intensity Level Slicing
  - Highlighting specific range of intensities in an image.
  - Enhances features such as masses of water in satellite imagery and enhancing flaws in X-ray images.
  - It can be Implemented two ways:
  - 1) To display only one value (say, white) in the range of interest and rests are black which produces binary image.
  - 2) brightens (or darkens) the desired range of intensities but leaves all other intensity levels in the image unchanged.

Piecewise-Linear Transformation Functions

- Intensity Level Slicing

a b

FIGURE 3.11 (a) This transformation highlights intensity range [*A*, *B*] and reduces all other intensities to a lower level. (b) This transformation highlights range [*A*, *B*] and preserves all other intensity levels.



Piecewise-Linear Transformation Functions

- Intensity Level Slicing

a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Piecewise-Linear Transformation Functions

- Bit Plane Slicing
  - Pixels are digital numbers composed of bits.
  - 256 gray scale image is composed of 8 bits.
  - Instead of highlighting intensity level ranges, we could highlight the contribution made to total image appearance by specific bits.
  - 8-bit image may be considered as being composed of eight 1-bit planes, with plane 1 containing the lowest- order bit of all pixels in the image and plane 8 all the highest-order bits.

Piecewise-Linear Transformation Functions

- Bit Plane Slicing

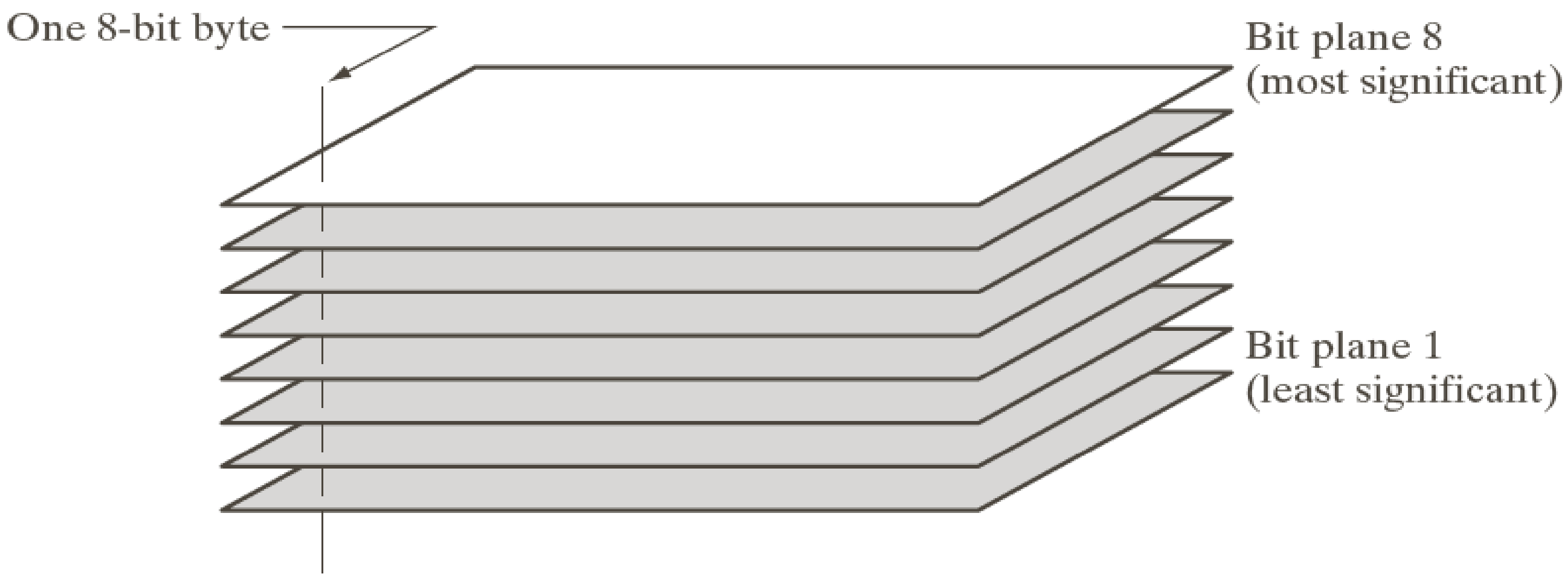


FIGURE 3.13 Bit-plane representation of an 8-bit image.

Piecewise-Linear Transformation Functions

- Bit Plane Slicing

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Piecewise-Linear Transformation Functions

- Bit Plane Slicing

a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Histogram Processing

- Histogram of a digital image with intensity levels in the range [0,L-1] is a discrete function $h(r_k) = n_k$, where r_k is the kth intensity value and n_k is the number of pixels in the image with intensity r_k
- Normalized histogram $p(r_k)=n_k/MN$, for k = 0,1,2……. L-1.
- Histogram manipulation can be used for image enhancement.
- Information inherent in histogram also is quite useful in other image processing applications, such as image compression and segmentation.

Histogram Equalization

- Intensity mapping form

$$s = T(r),\ 0 \le r \le L-1$$

Conditions:

a) T(r) is a monotonically increasing function in the interval [0, L-1] and

b) $0 \le T(r) \le L-1 \; for 0 \le r \le L-1$

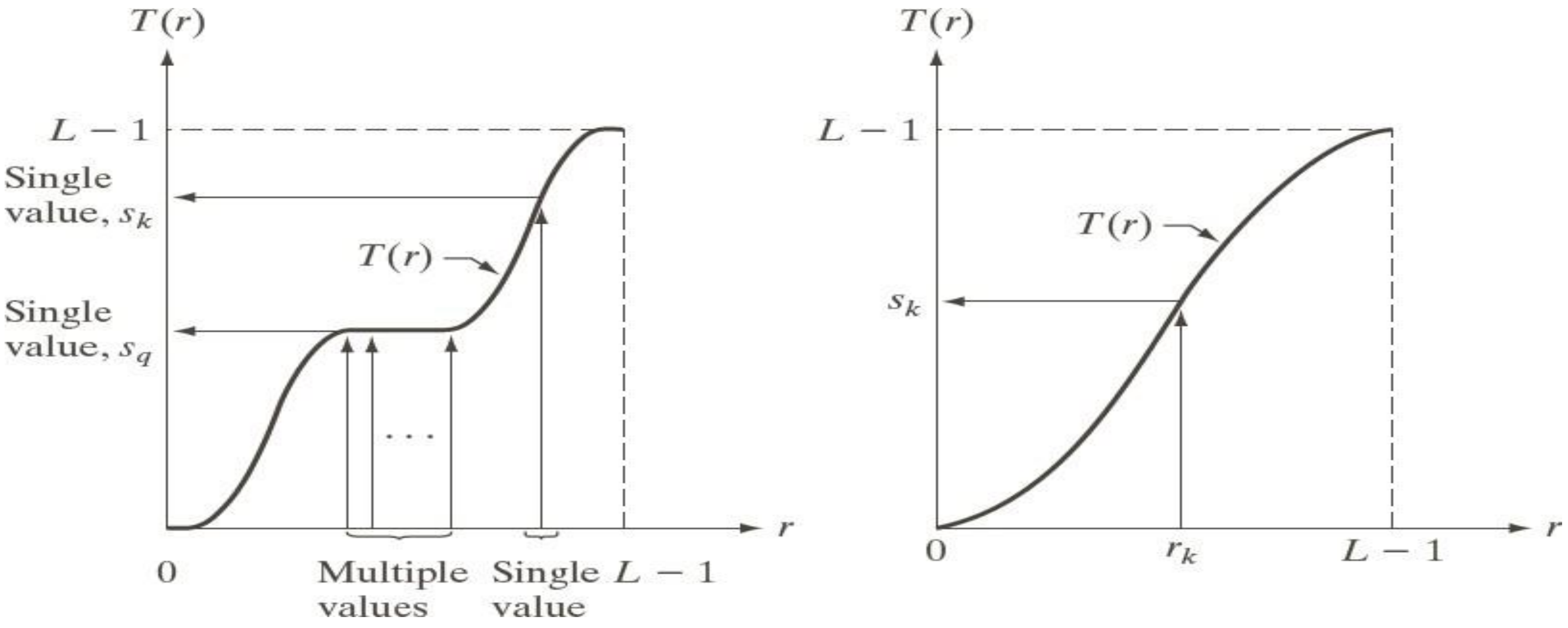
In some formulations, we use the inverse in which case

$$r = T^{-1}(s), \quad 0 \le s \le 1$$

(a) change to

a') T(r) is a strictly monotonically increasing function in the interval [0, L-1]

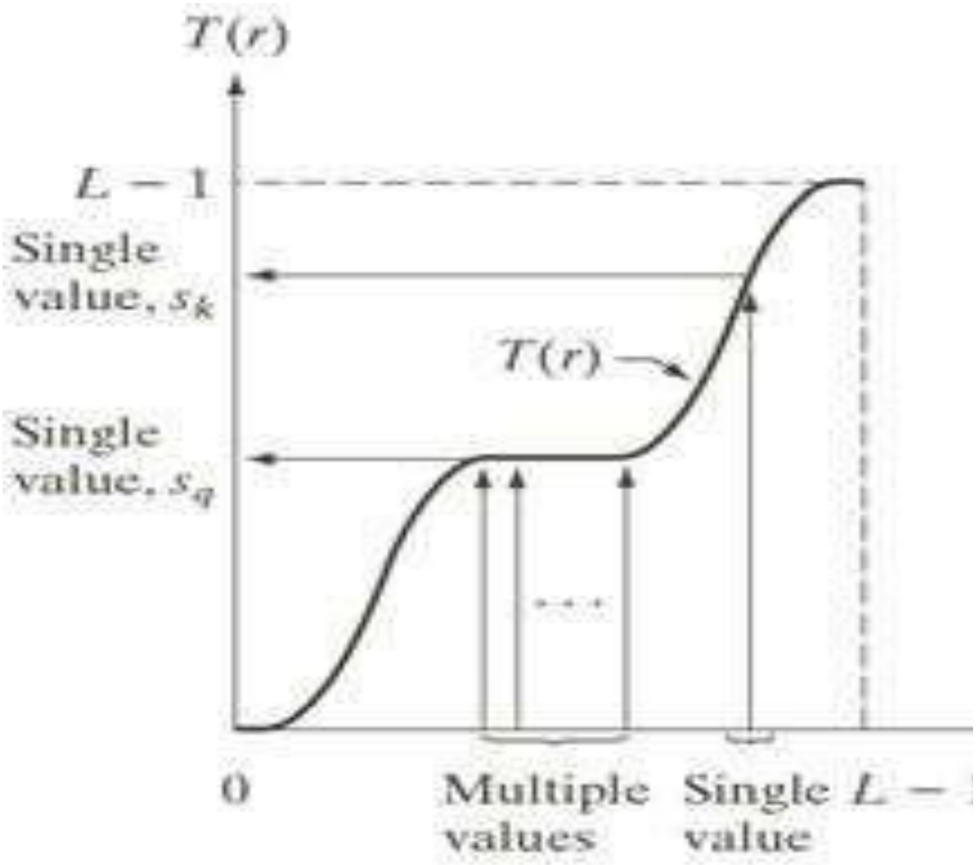
Histogram Processing

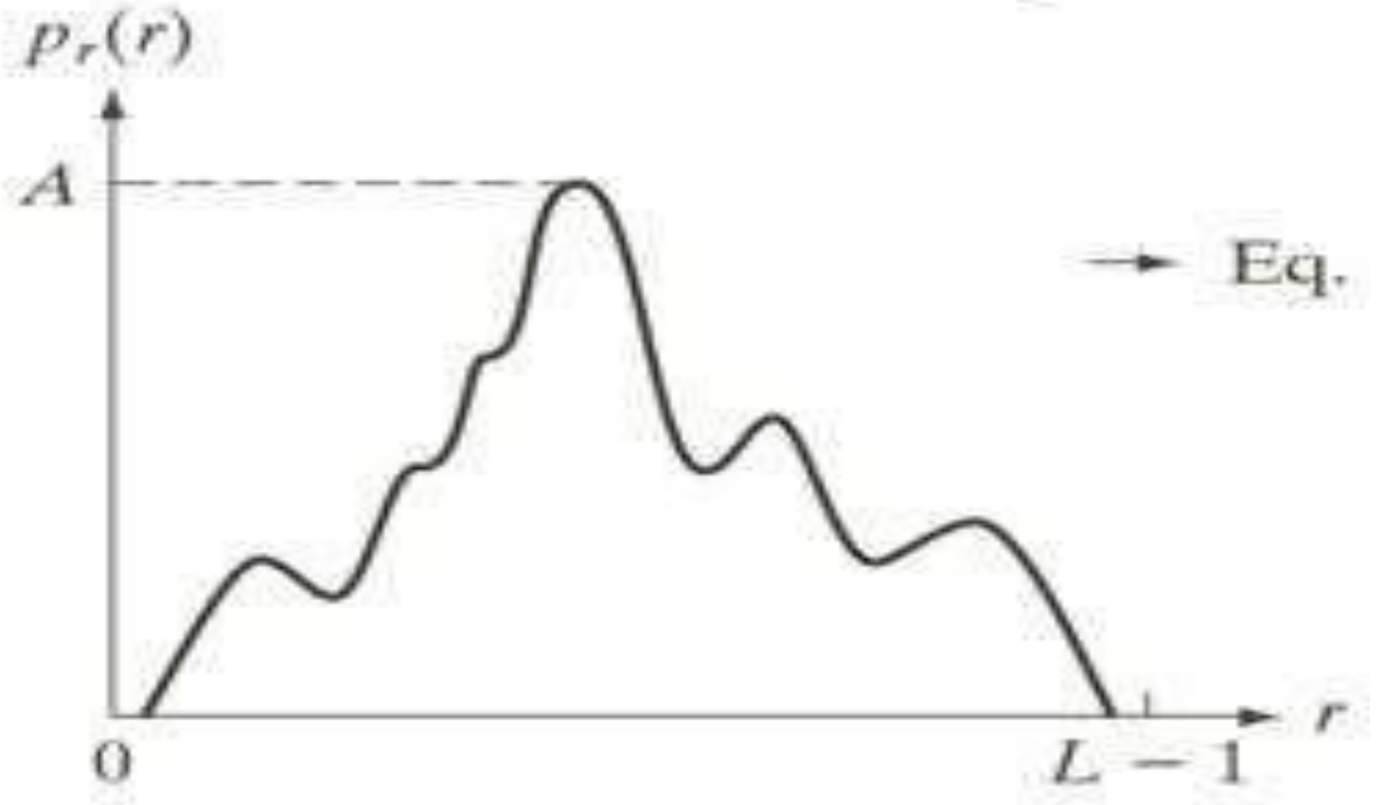


a b

FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Processing





Histogram Equalization

- Intensity levels in an image may be viewed as random variables in the interval [0,L-1]
- Fundamental descriptor of a random variable is its probability density function (PDF)

- Let $p_r(r)$ and $p_s(s)$ denote the PDFs of r and s respectively

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)dw\right] = (L-1)p_r(r)$$

Histogram Processing

$$p_s(s) = \frac{1}{L-1}$$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k} \frac{n_j}{MN}, \quad k = 0,1,2,\ldots,L-1$$

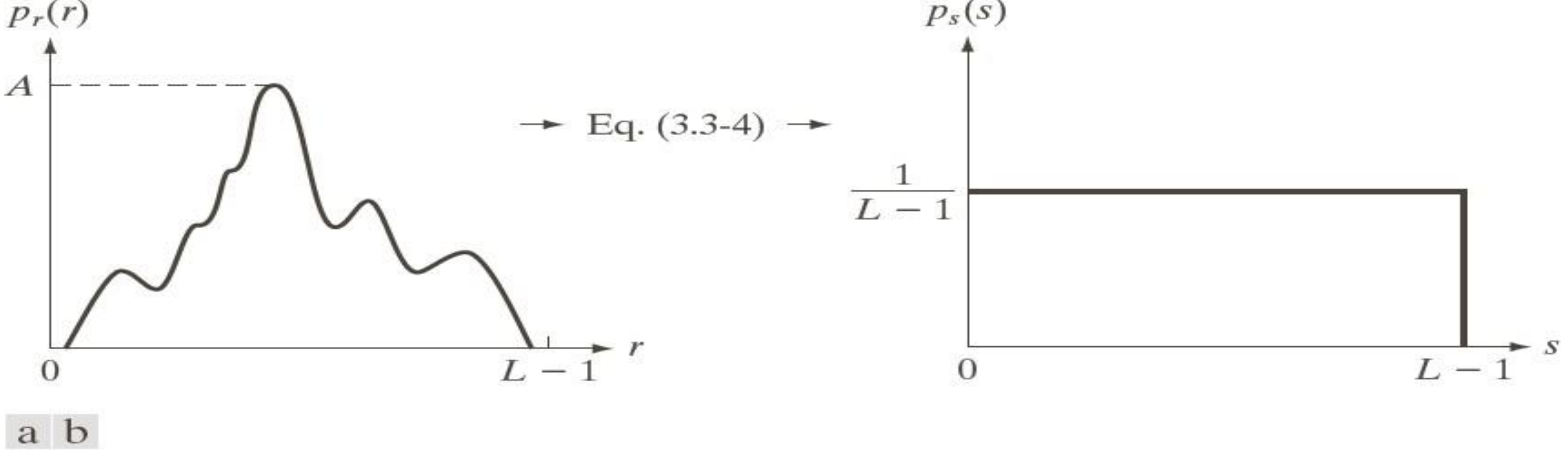
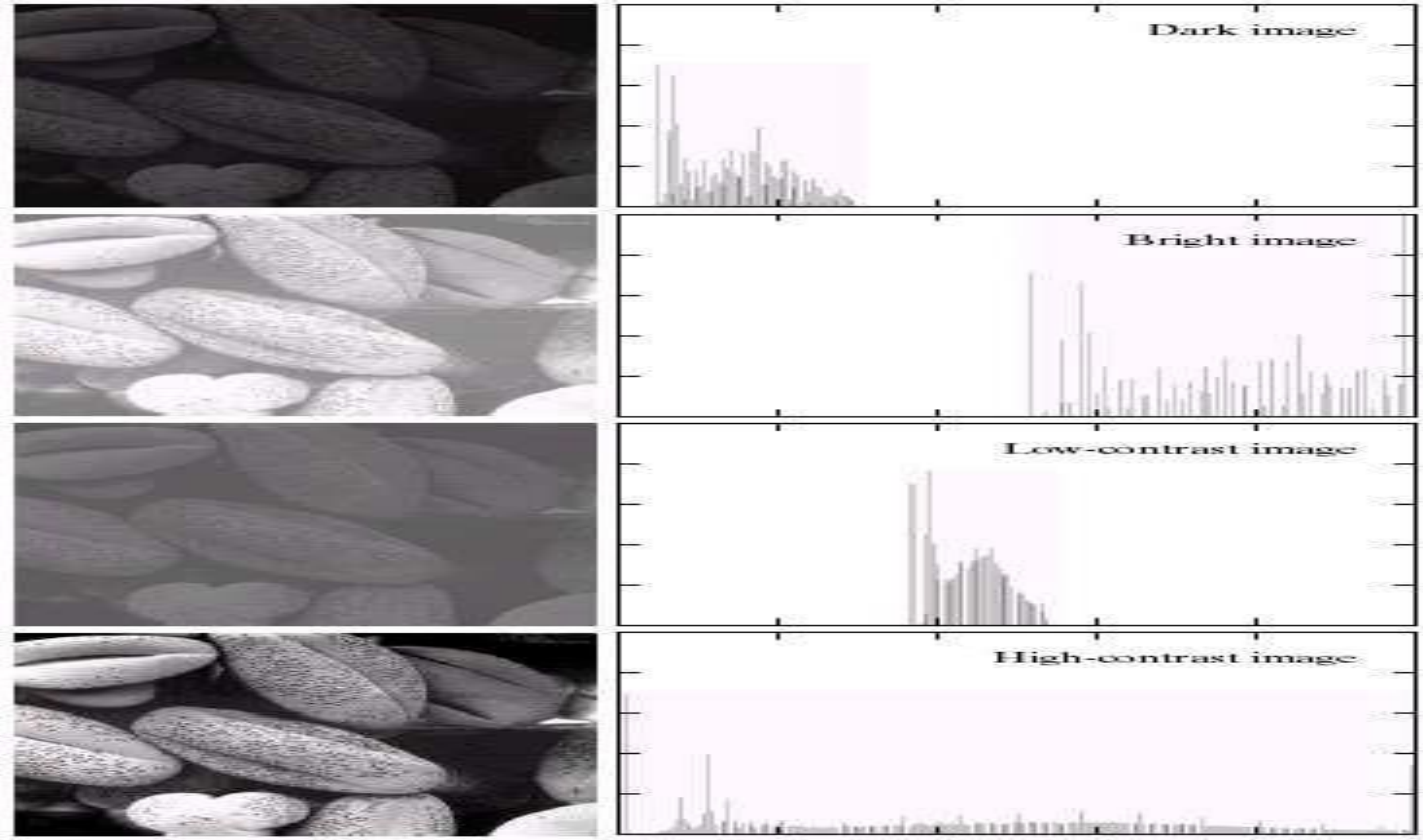


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Processing



Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1
Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

Histogram Processing

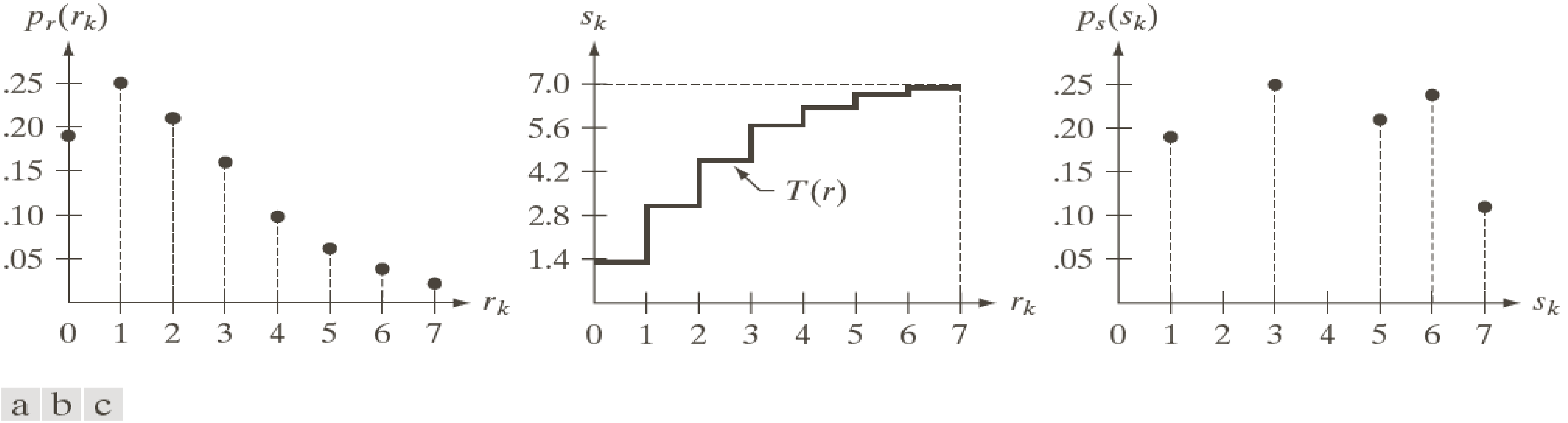


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Processing

- Transformation functions

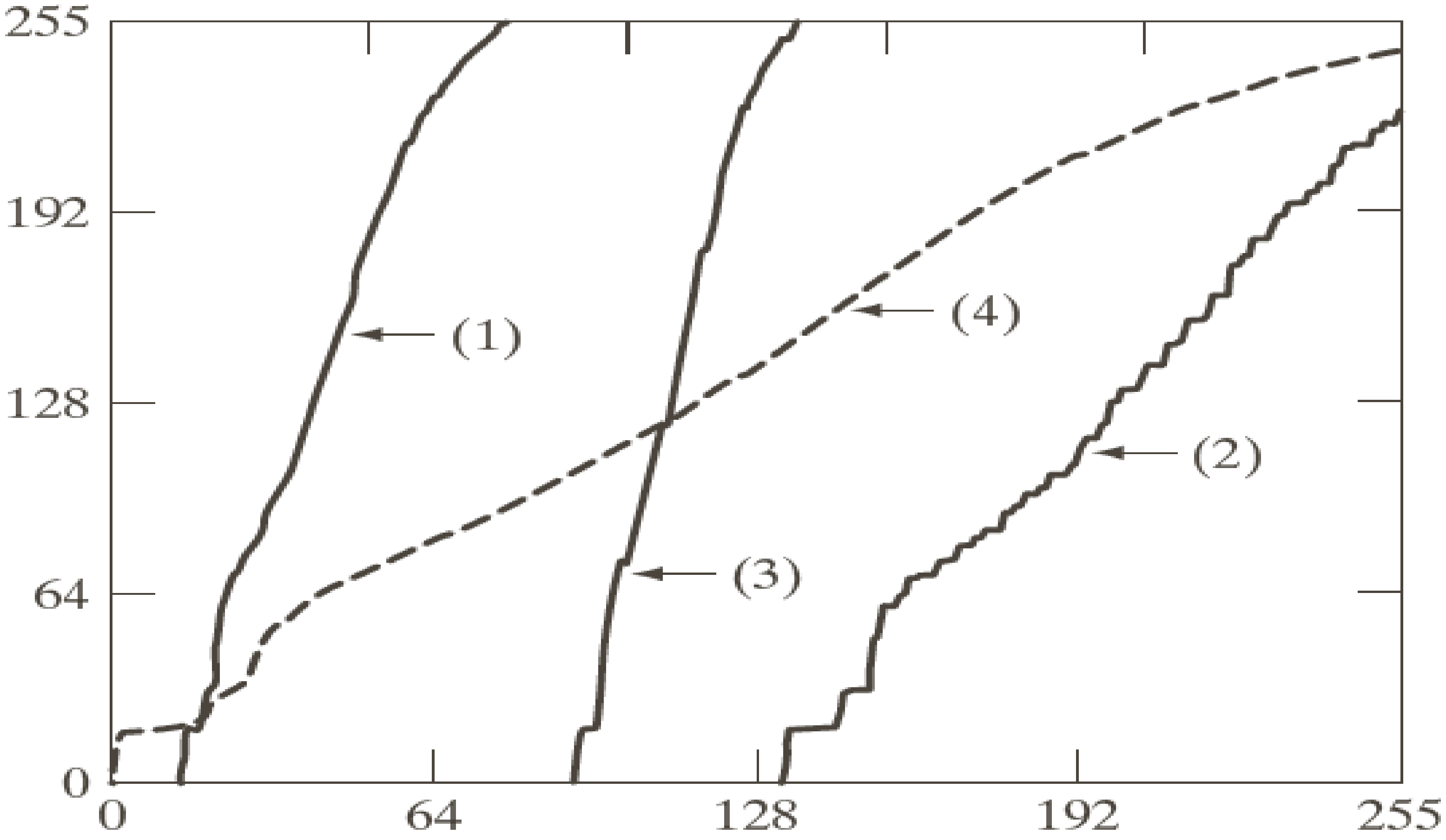


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

Histogram Equalization

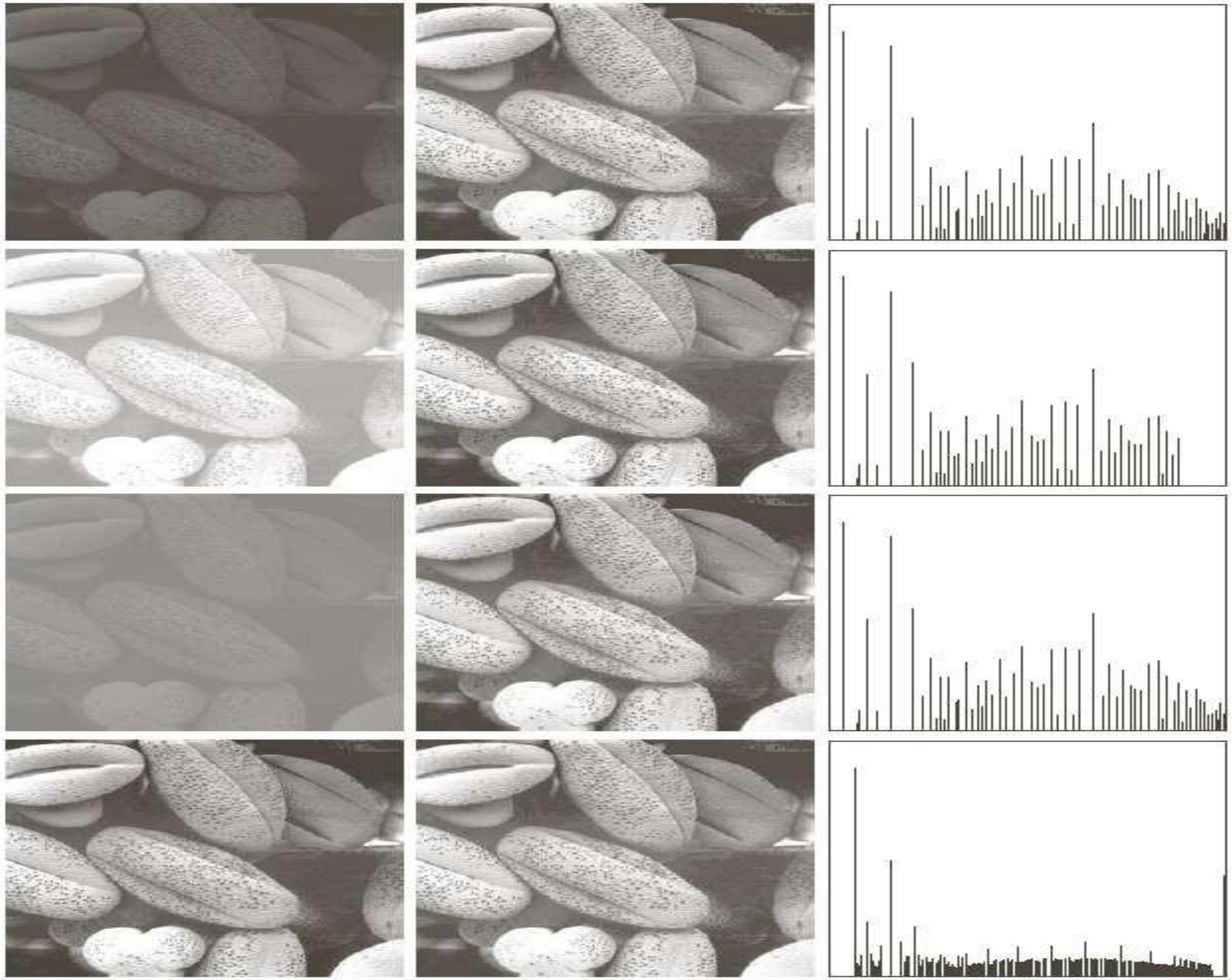
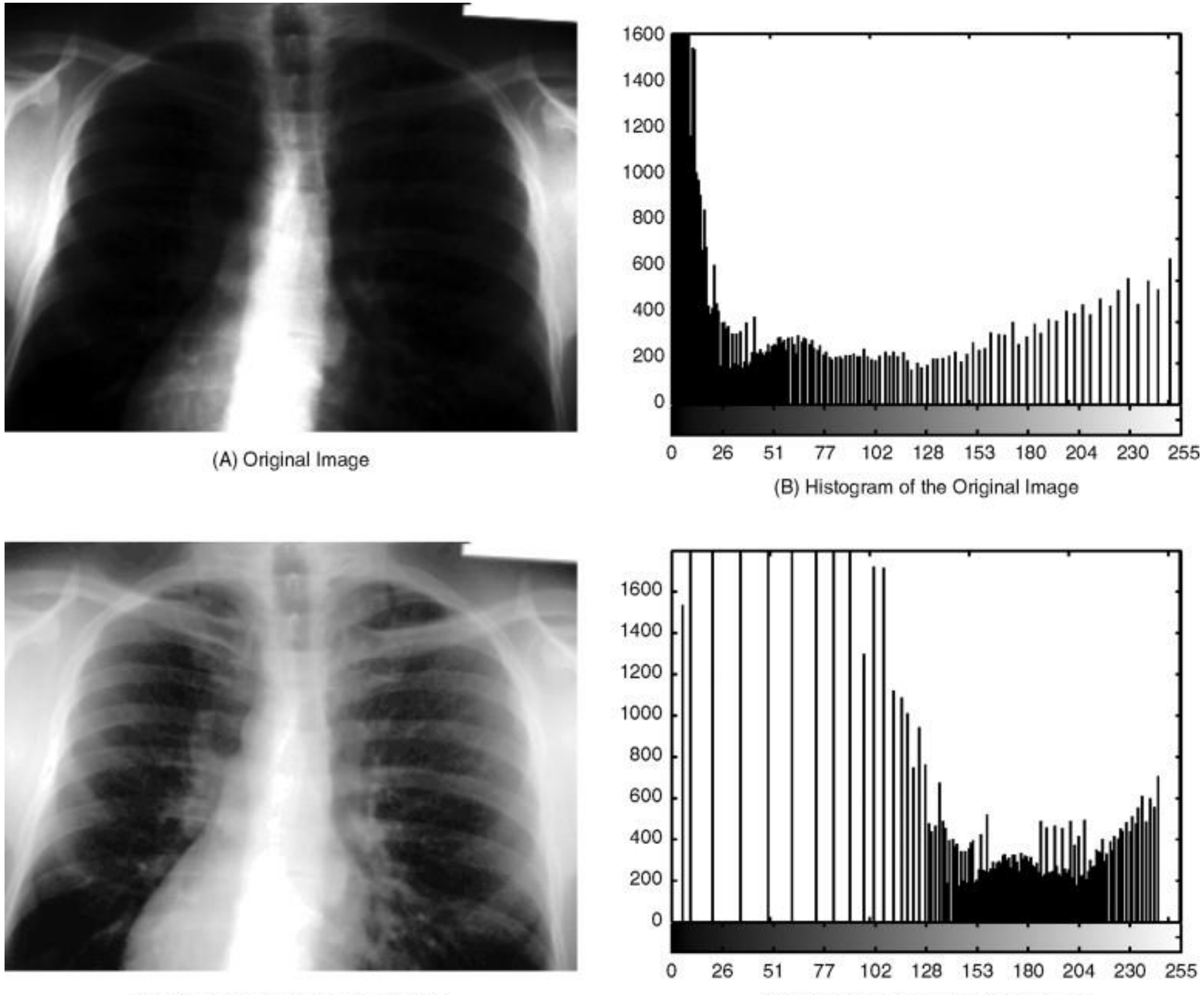


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Histogram Equalization



(A) Original Image

(B) Histogram of the Original Image

(C) Image with Equalized Histogram

(D) Histogram of the Equalized Image

Histogram Matching (Specification)

- Histogram equalization automatically determines a transformation function produce uniform histogram
- When automatic enhancement is desired, equalization is a good approach
- There are some applications in which attempting to base enhancement on a uniform histogram is not the best approach
- In particular, it is useful sometimes to be able to specify the shape of the histogram that we wish the processed image to have.
- The method used to generate a processed image that has a specified histogram is called *histogram matching* or *specification*

Histogram Matching (Specification)

Histogram Specification Procedure:

1) Compute the histogram $p_r(r)$ of the given image, and use it to find the histogram equalization transformation in equation $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} \frac{n_j}{MN}, \quad k = 0,1,2,\ldots,L-1$

and round the resulting values to the integer range [0, L-1]

2) Compute all values of the transformation function G using same equation $G(z_q) = (L-1)\sum_{i=0}^{q} p_z(r_i), \quad q = 0,1,2,\ldots, L-1$

and round values of G

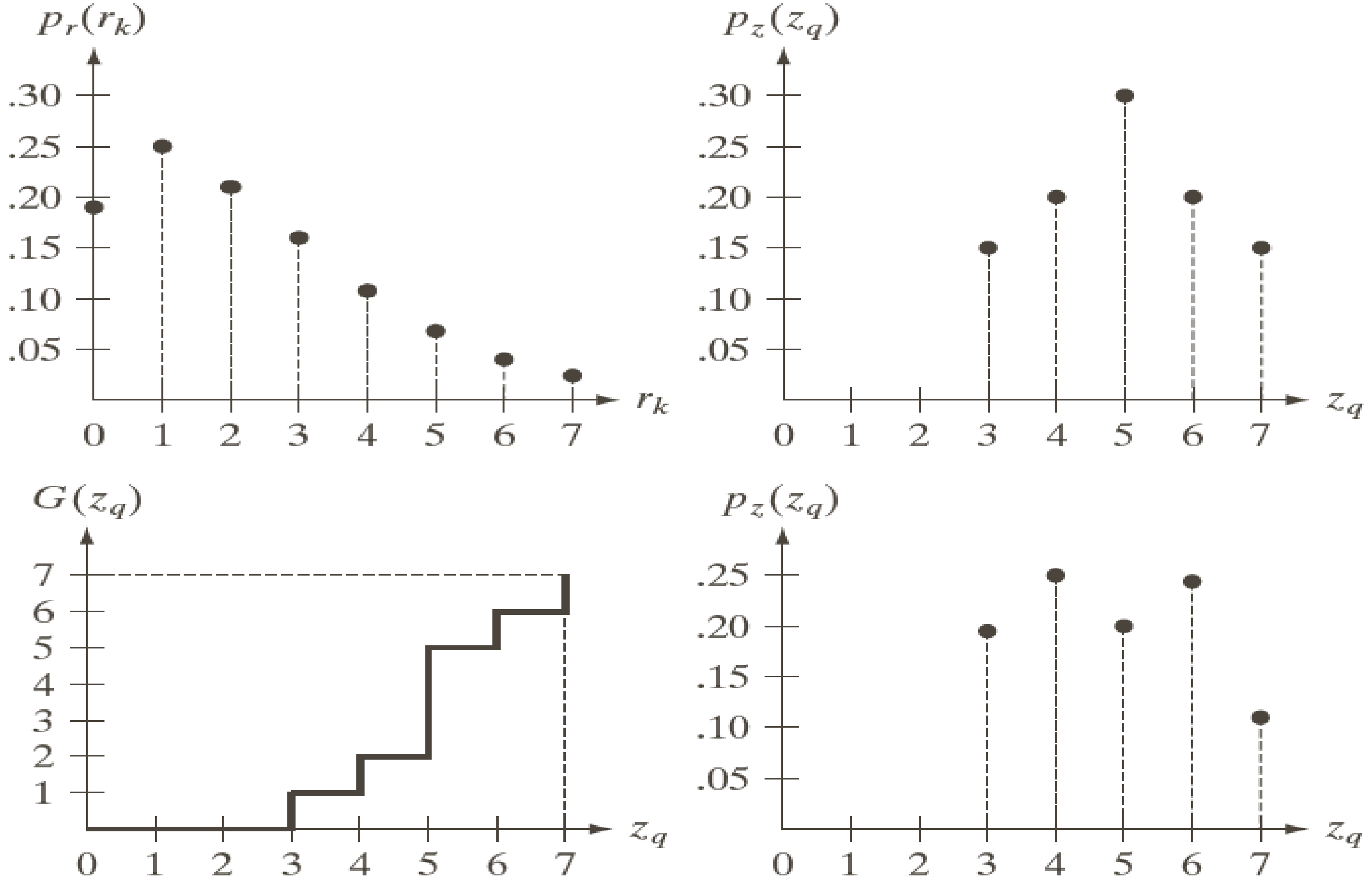
3) For every value of s_k, k = 0,1,...,L-1, use the stored values of G to find the corresponding value of z_q so that $G(z_q)$ is closet to s_k and store these mappings from s to z.

Histogram Matching (Specification)

- Histogram Specification Procedure:

4) Form the histogram-specified image by first histogram- equalizing the input image and then mapping every equalized pixel value, s_k, of this image to the corresponding value z_q in the histogram-specified image using the mappings found in step 3.

Histogram Matching



a	b
c	d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Histogram Matching

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2
Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

Histogram Matching

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

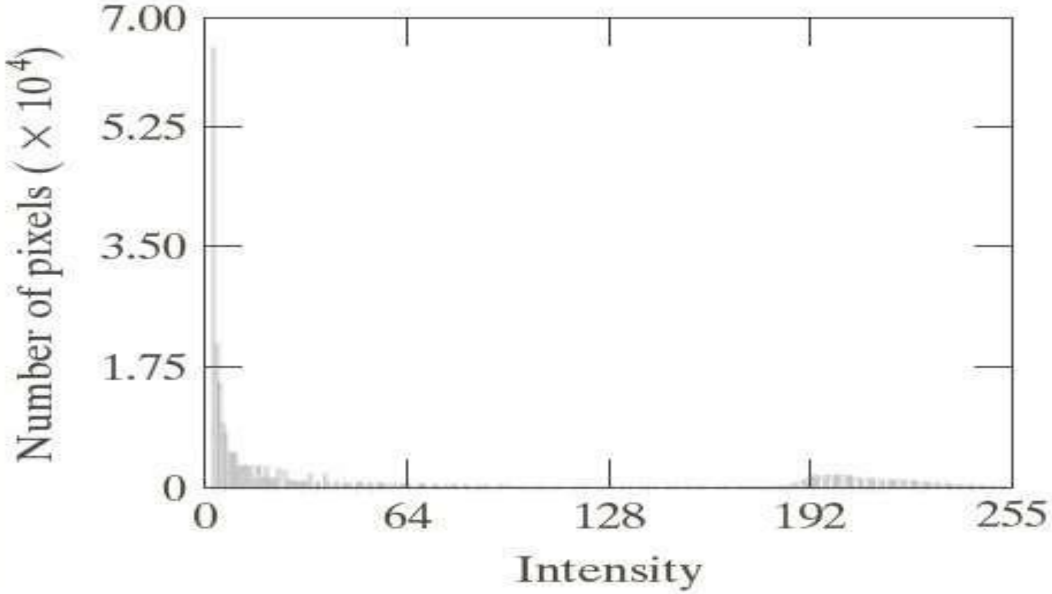
TABLE 3.3
All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

r_k	s_k	$G(Z_q)$
0	1	0
1	3	0
2	5	0
3	6	1
4	6	2
5	7	5
6	7	6
7	7	7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

TABLE 3.4
Mappings of all the values of s_k into corresponding values of z_q.

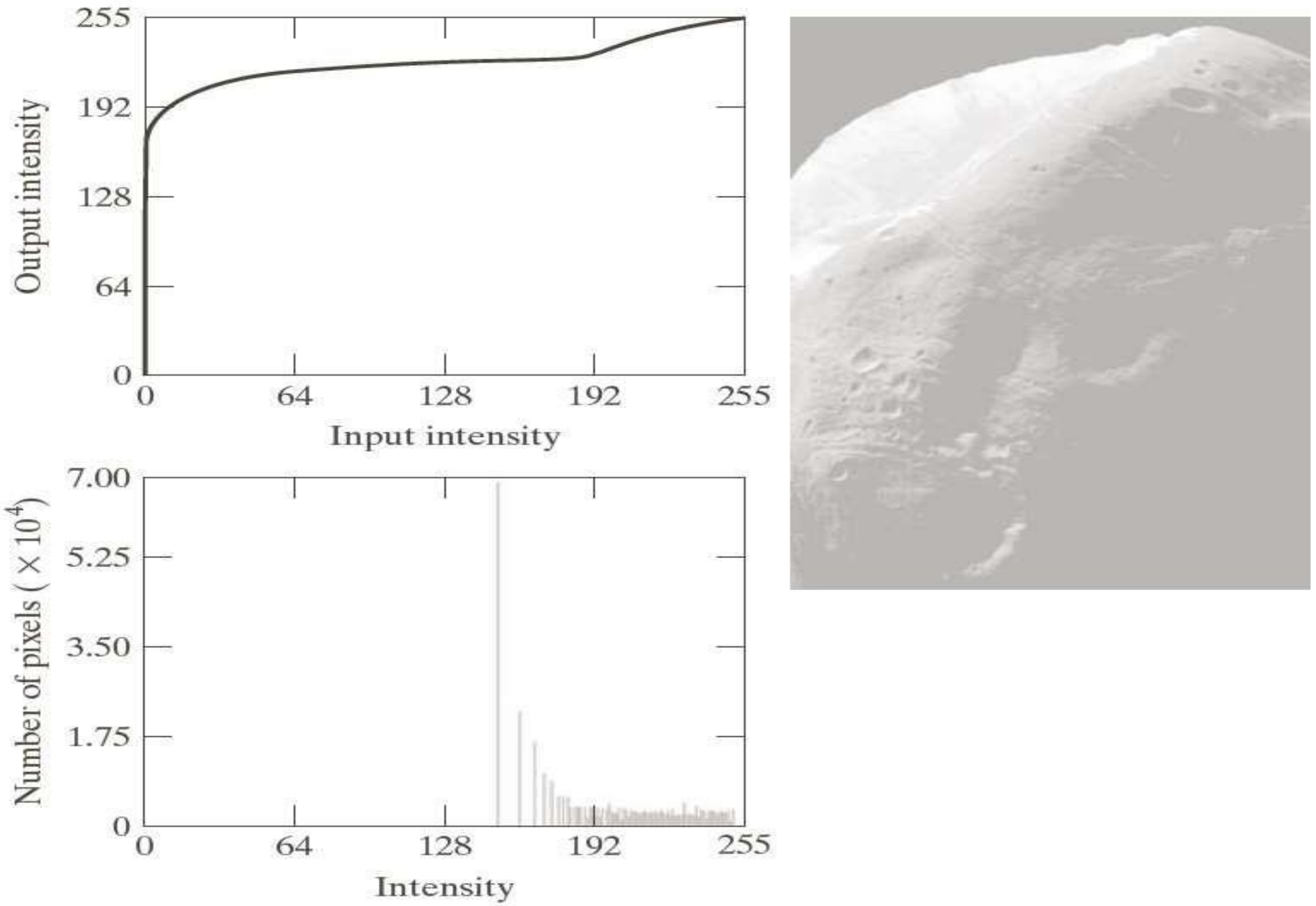
Histogram Matching



a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

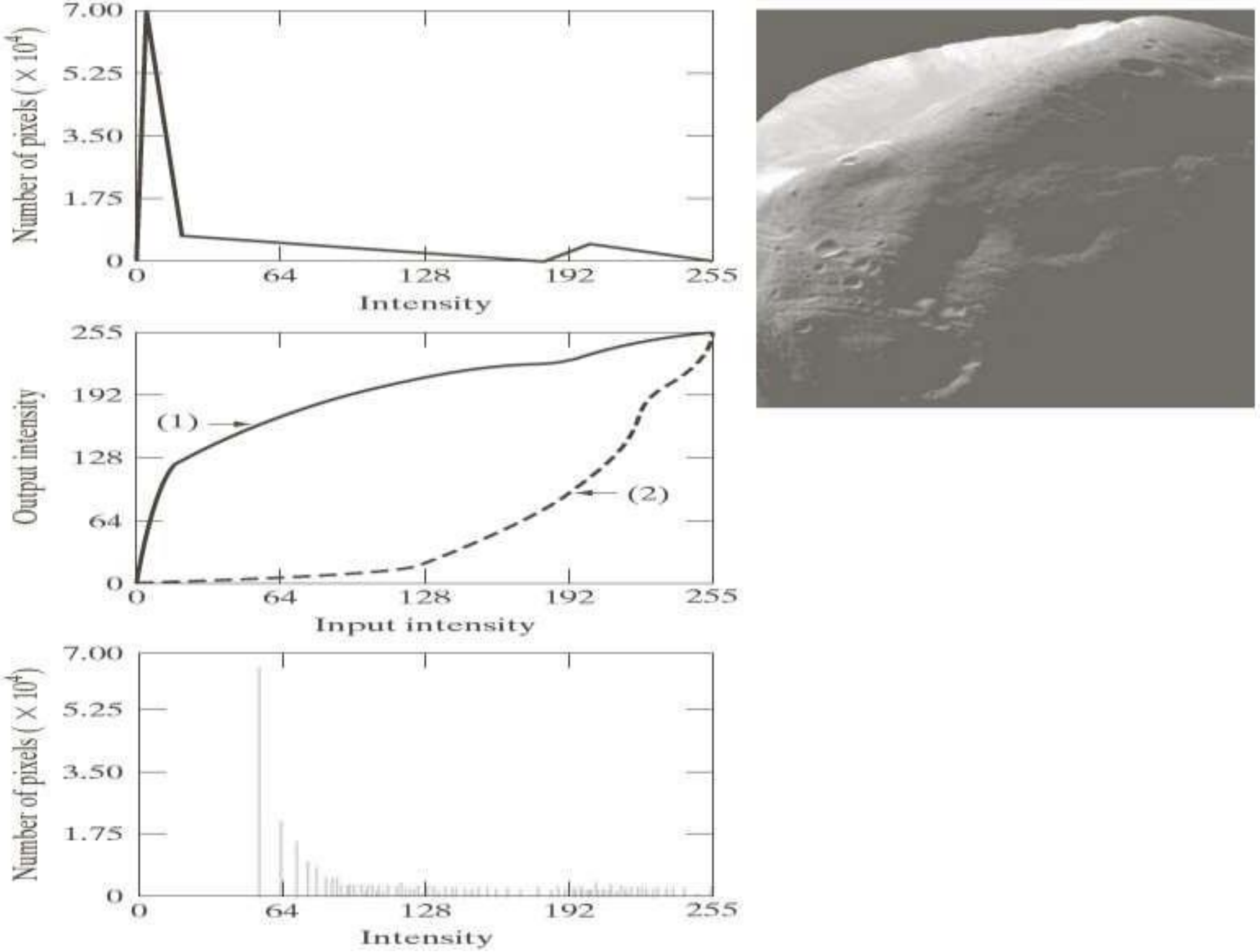
Histogram Matching



a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

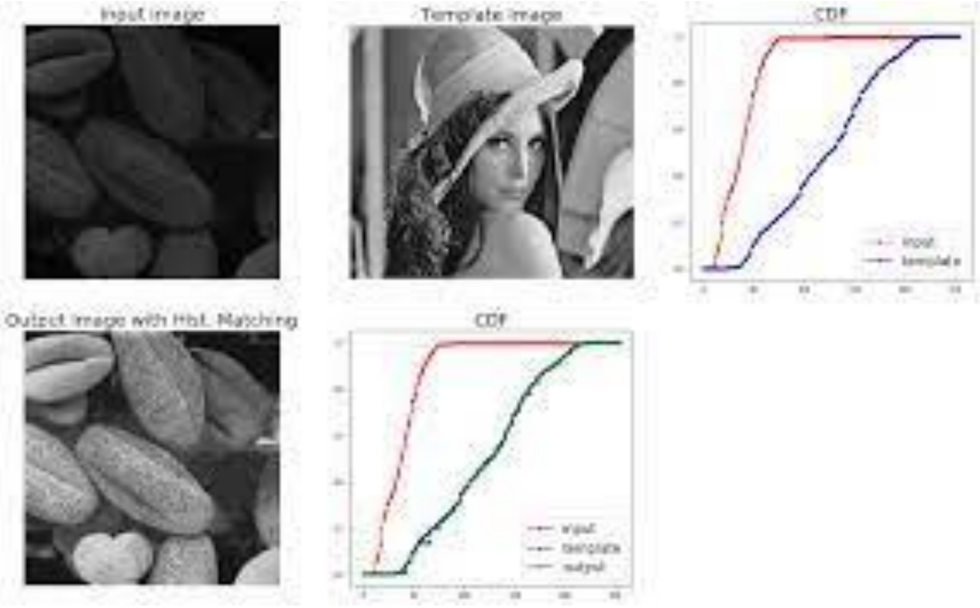
Histogram Matching

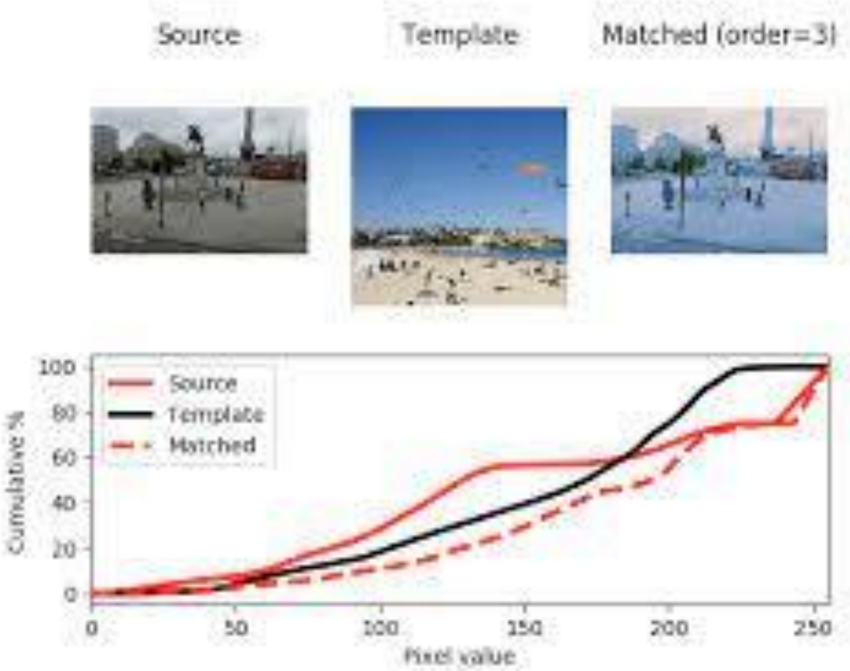


a	c
b	
d	

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

Histogram Matching





Local Histogram Processing

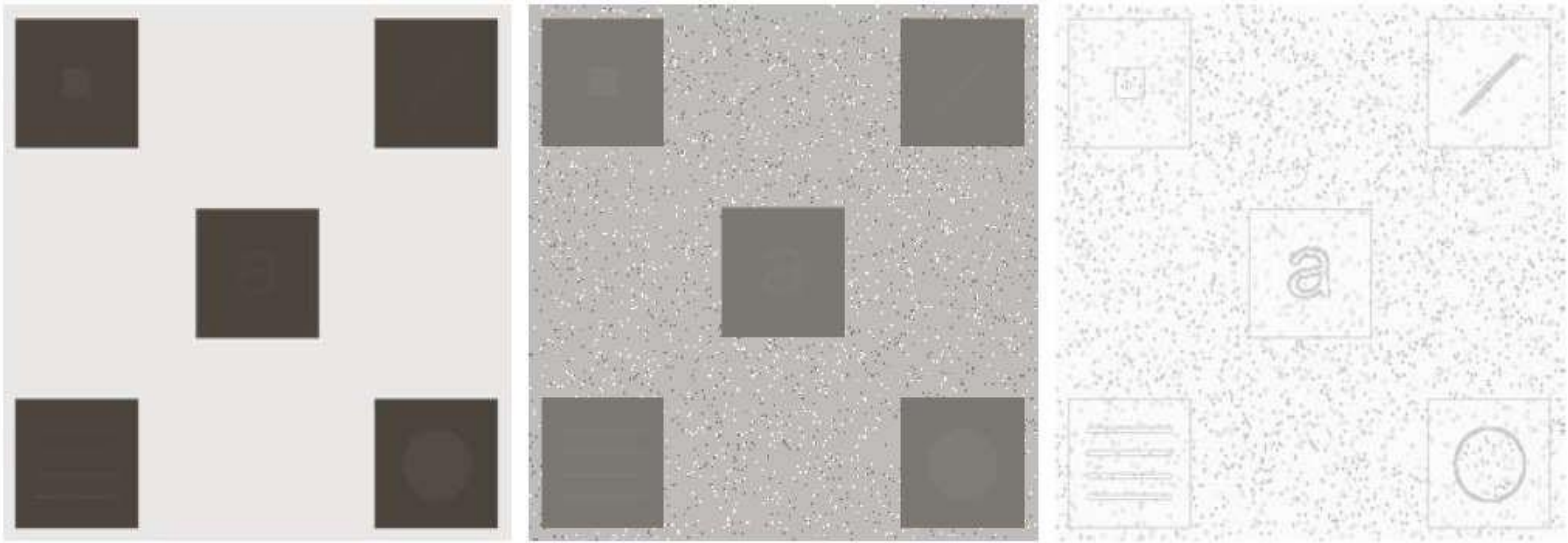
- Histogram Processing methods discussed in the previous two sections are *Global,* in the sense that pixels are modified by a transformation function based on the intensity distribution of an entire image.
- There are some cases in which it is necessary to enhance detail over small areas in an image.
- This procedure is to define a neighborhood and move its center pixel to pixel.
- At each location, the histogram of the points in the neighborhood is computed and either a histogram equalization or histogram specification transformation function is obtained.
- Map the intensity of the pixel centered in the neighborhood.
- Center of the neighborhood region is then moved to an adjacent pixel location and the procedure is repeated.

Local Histogram Processing

- This approach has obvious advantages over repeatedly computing the histogram of all pixels in the neighborhood region each time the region is moved one pixel location.

- Another approach used sometimes to reduce computation is to utilize non overlapping regions, but this method usually produces an undesirable *"blocky"* effect.

Local Histogram Processing



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Spatial Filtering

- Also called spatial masks, kernels, templates, and windows.
- It consists of (1) a neighborhood (typically a small window), and (2) a predefined operation that is performed on the image pixels encompassed by the neighborhood.
- Filtering creates a new pixel with coordinates equal to the center of the neighborhood.
- If operation is linear, then filter is called a *linear spatial filter* otherwise *nonlinear.*

Mechanics of Spatial Filtering

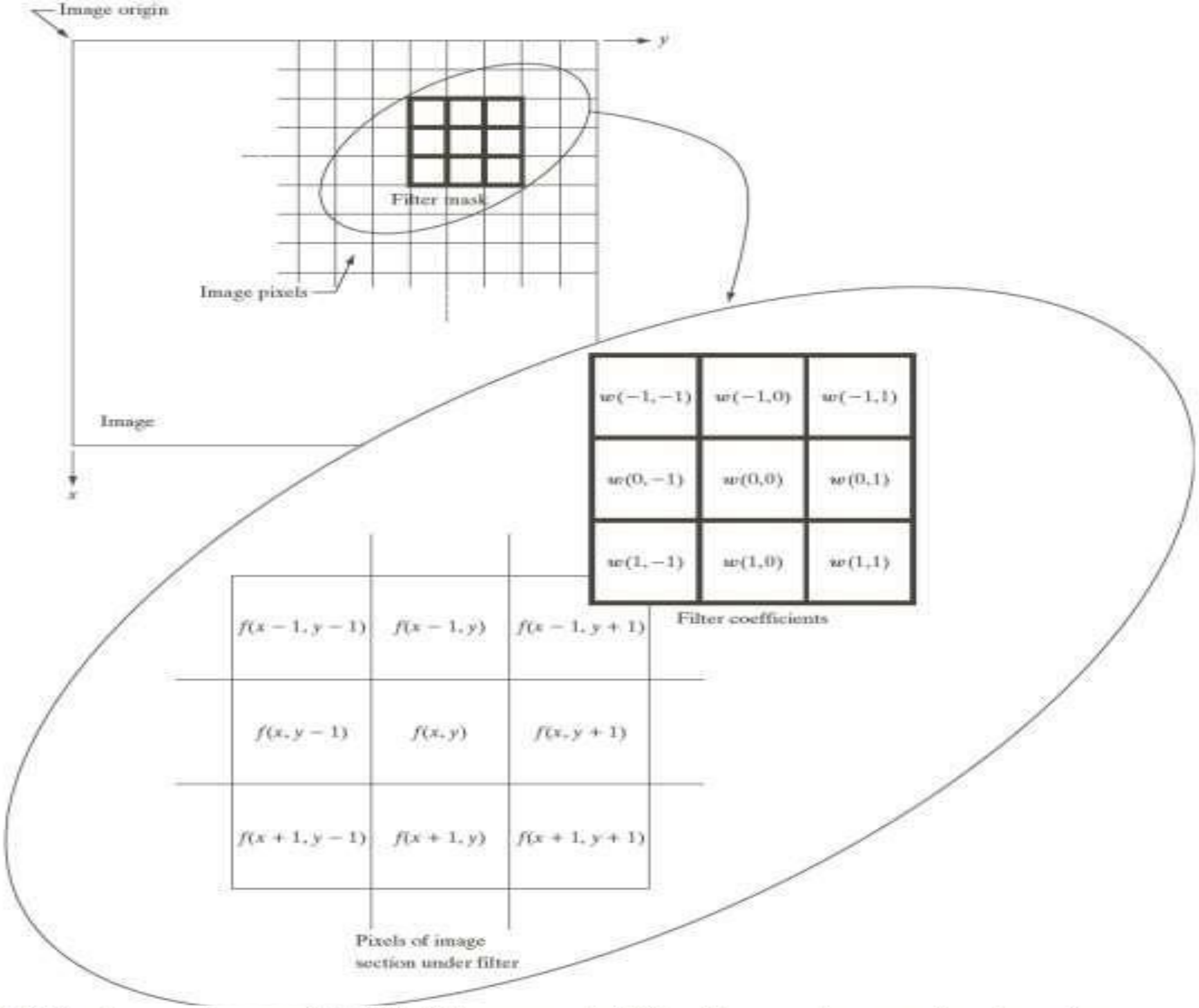


FIGURE 3.28 The mechanics of linear spatial filtering using a 3 × 3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

Spatial Correlation & Convolution

- Correlation is the process of moving a filter mask over the image and computing the sum of the products at each location.
- Convolution process is same except that the filter is first rotated by 180 degree.

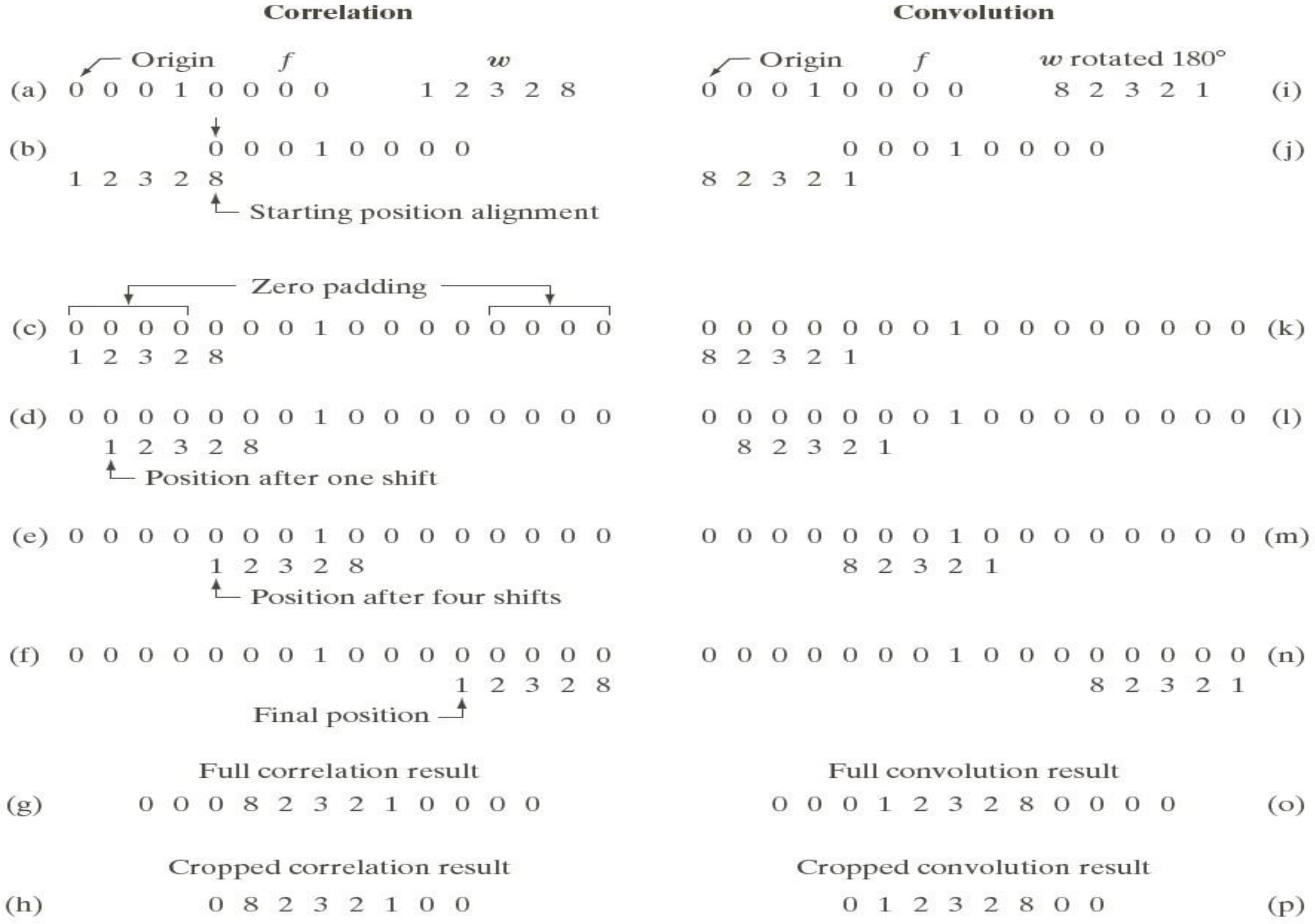


FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

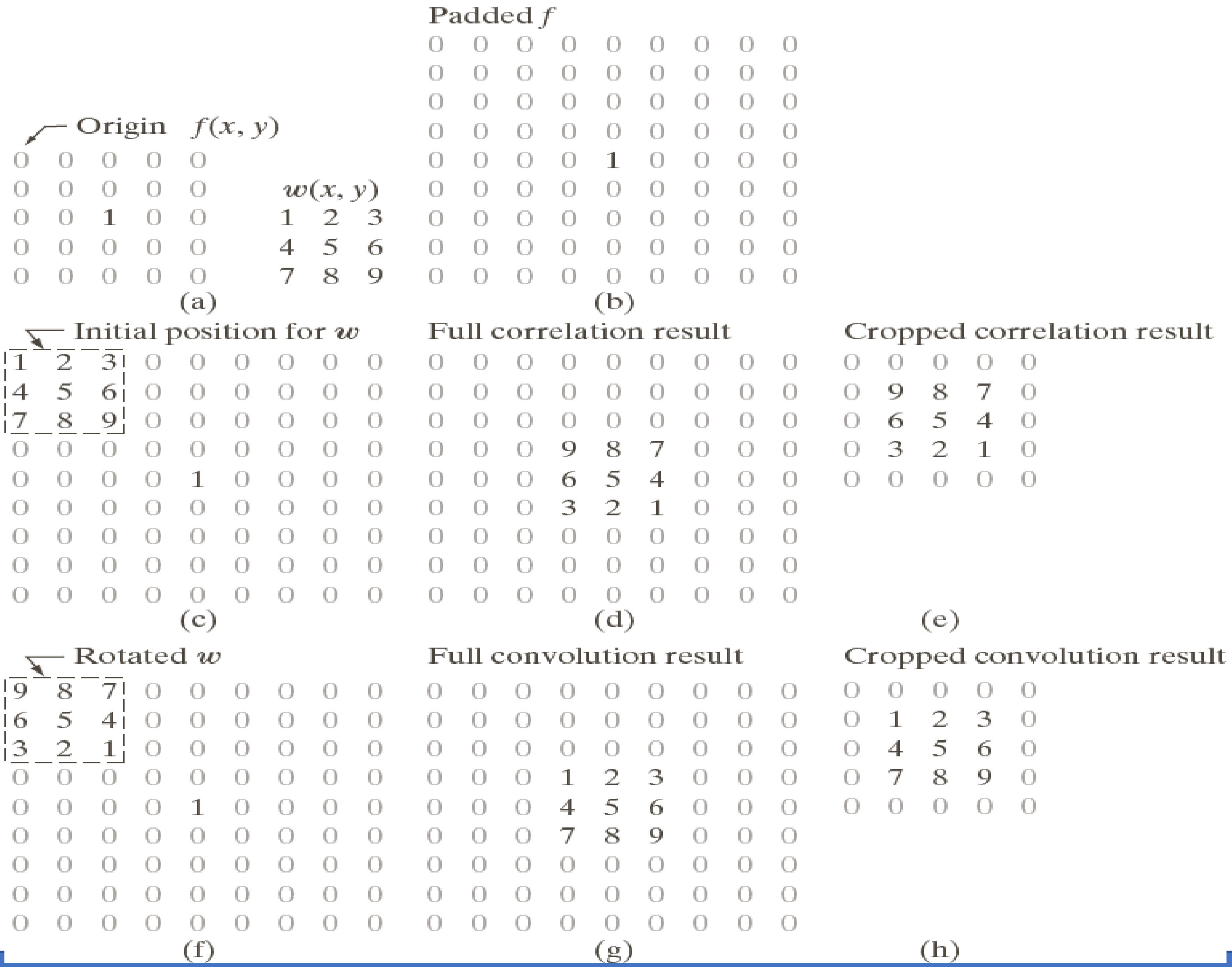


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

Smoothing Spatial Linear Filters

- Also called *averaging filters* or *Lowpass filter.*
- By replacing the value of every pixel in an image by the average of the intensity levels in the neighborhood defined by the filter mask.
- Reduced “sharp” transition in intensities.
- Random noise typically consists of sharp transition.
- Edges also characterized by sharp intensity transitions, so averaging filters have the undesirable side effect that they blur edges.
- If all coefficients are equal in filter than it is also called *a box filter*.

Smoothing Spatial Linear Filters

The other mask is called *weighted average,* terminology used to indicate that pixels are multiplied by different coefficient.

- Center point is more weighted than any other points.
- Strategy behind weighing the center point the highest and then reducing value of the coefficients as a function of increasing distance from the origin is simply an attempt to reduce blurring in the smoothing process.
- Intensity of smaller object blends with background.

Smoothing Linear Filter

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

$$\frac{1}{16} \times$$

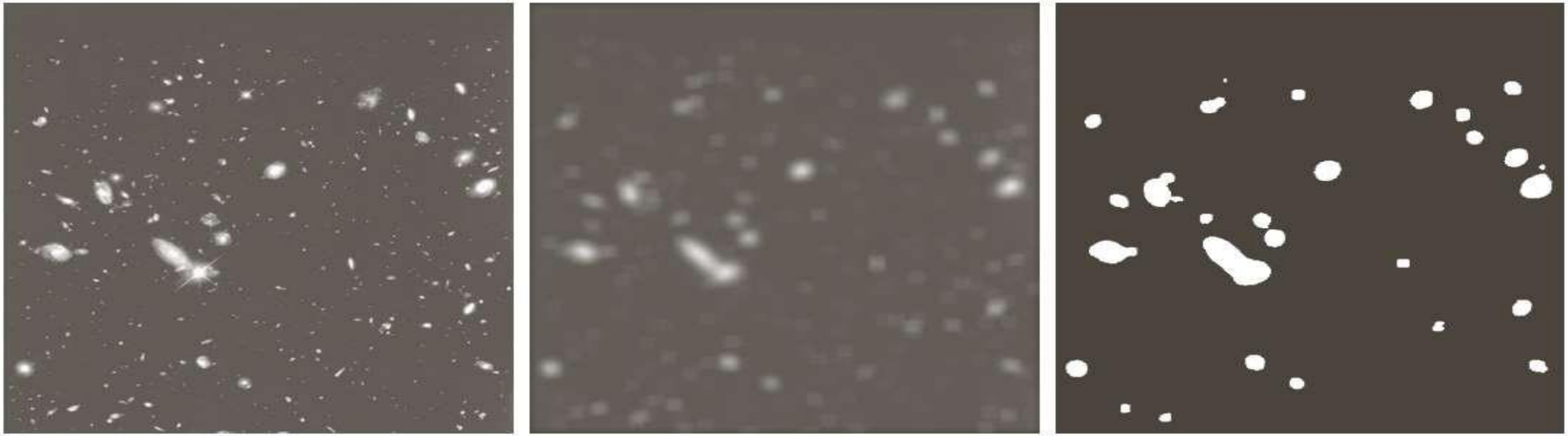
1	2	1
2	4	2
1	2	1

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

a	b
c	d
e	f

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes m = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.



a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Order-Statistic (Nonlinear) Filters

- Response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
- Best-known filter is *median filter.*
- Replaces the value of a center pixel by the median of the intensity values in the neighborhood of that pixel.
- Used to remove *impulse or salt-pepper noise*.
- Larger clusters are affected considerably less. ?
- Median represents the 50^{th} percentile of a ranked set of numbers while 100^{th} or 0^{th} percentile results in the so- called *max filter or min filter* respectively.

Median Filter (Nonlinear)

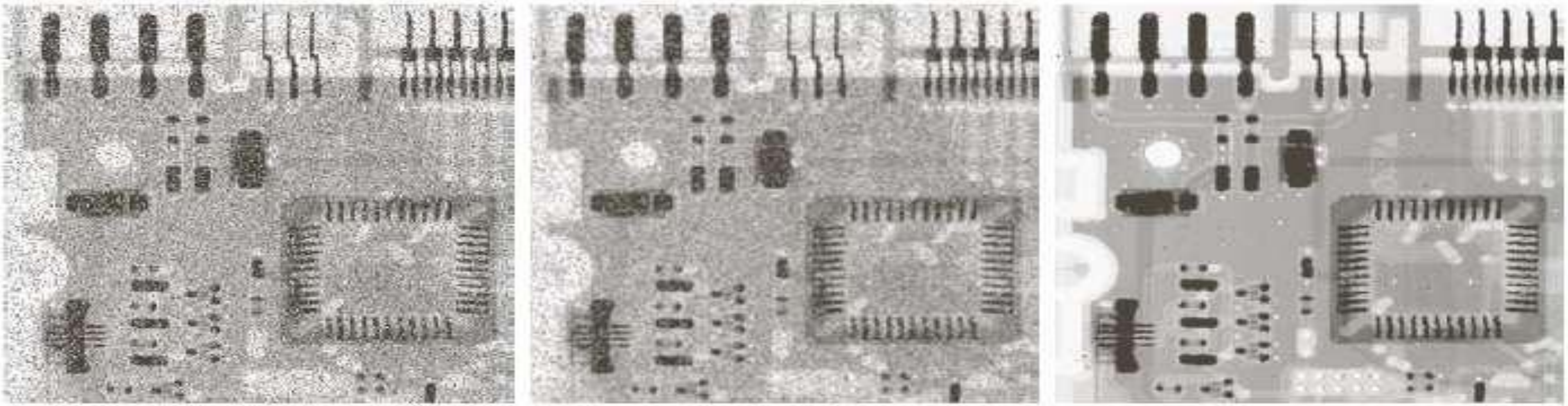
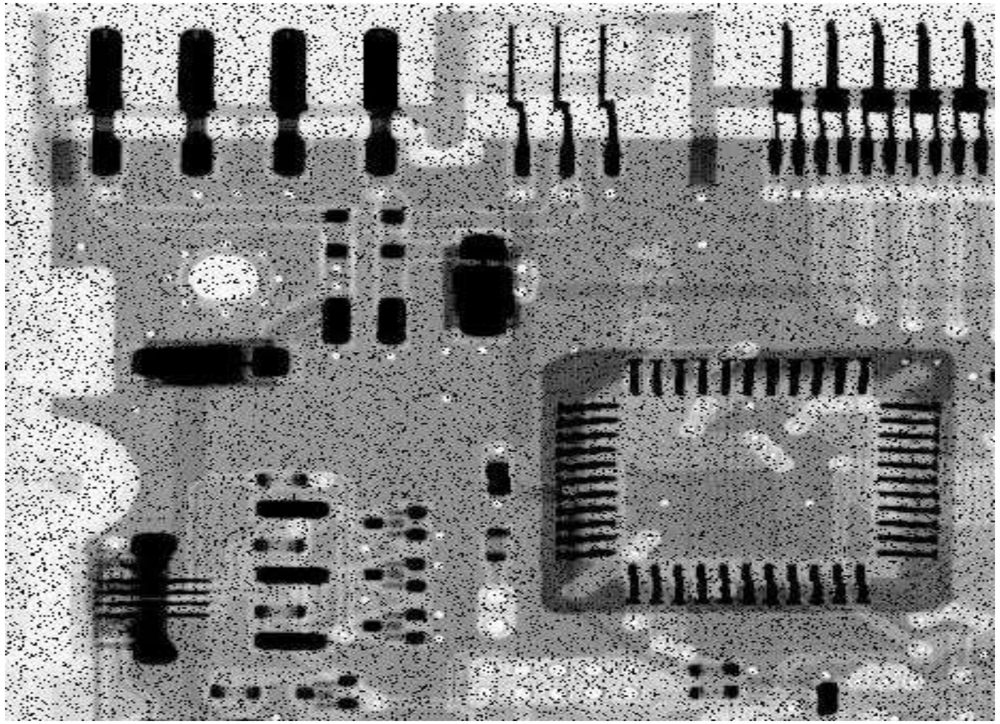
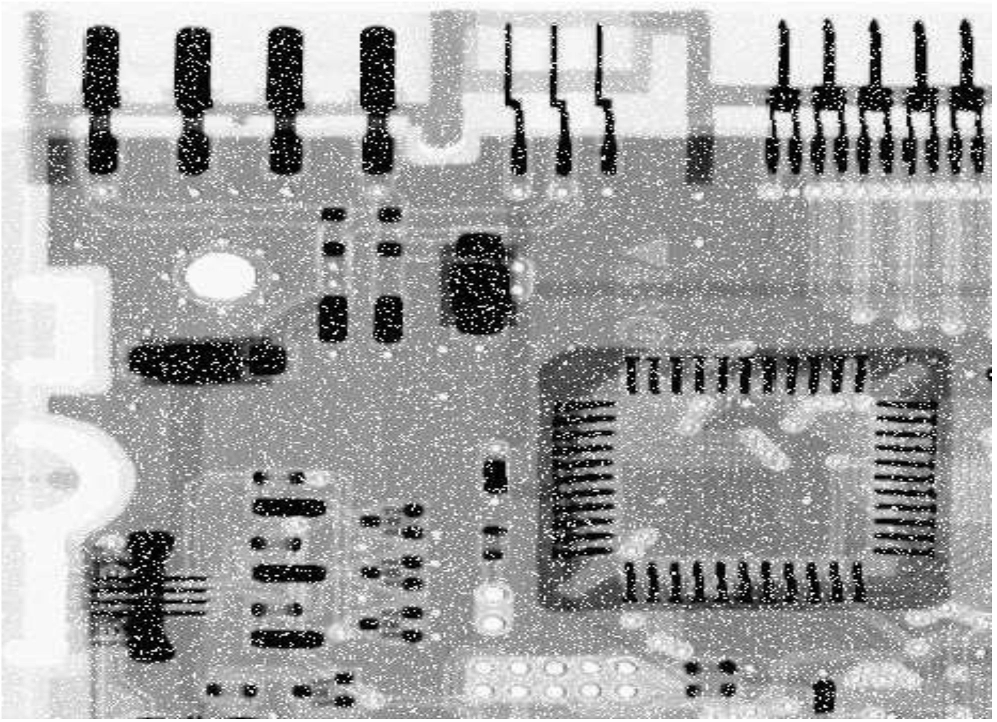


FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Median Filter (Nonlinear)





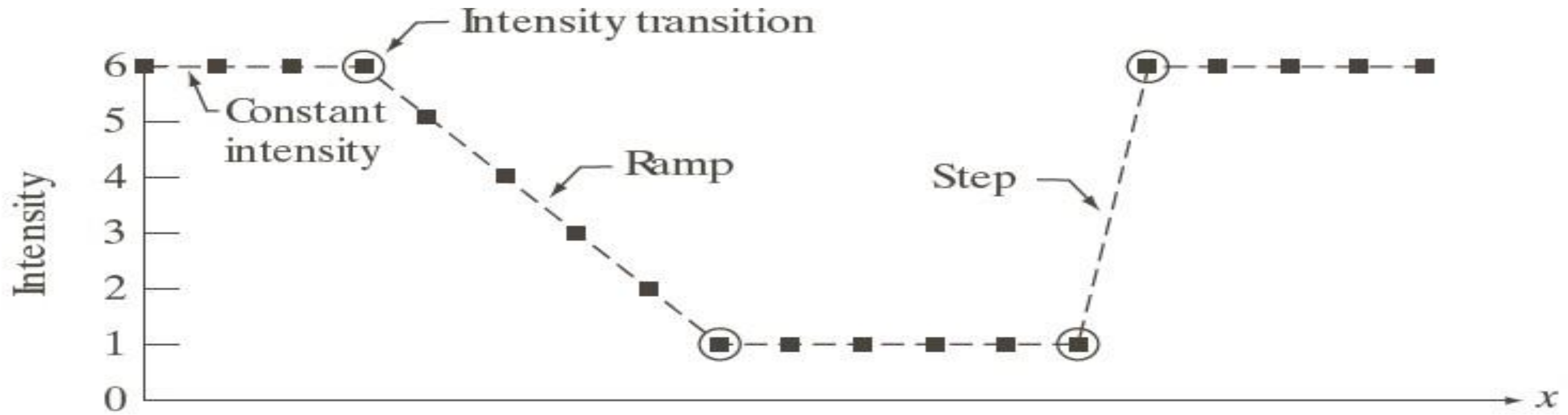
Sharpening Spatial Filters

- Objective of sharpening is to highlight transitions in intensity.
- Uses in printing and medical imaging to industrial inspection and autonomous guidance in military systems.

- Averaging is analogous to integration, so sharpening is analogous to spatial differentiation.
- Thus, image differentiation enhances edges and other discontinuities (such as noise) and deemphasizes areas with slowly varying intensities.

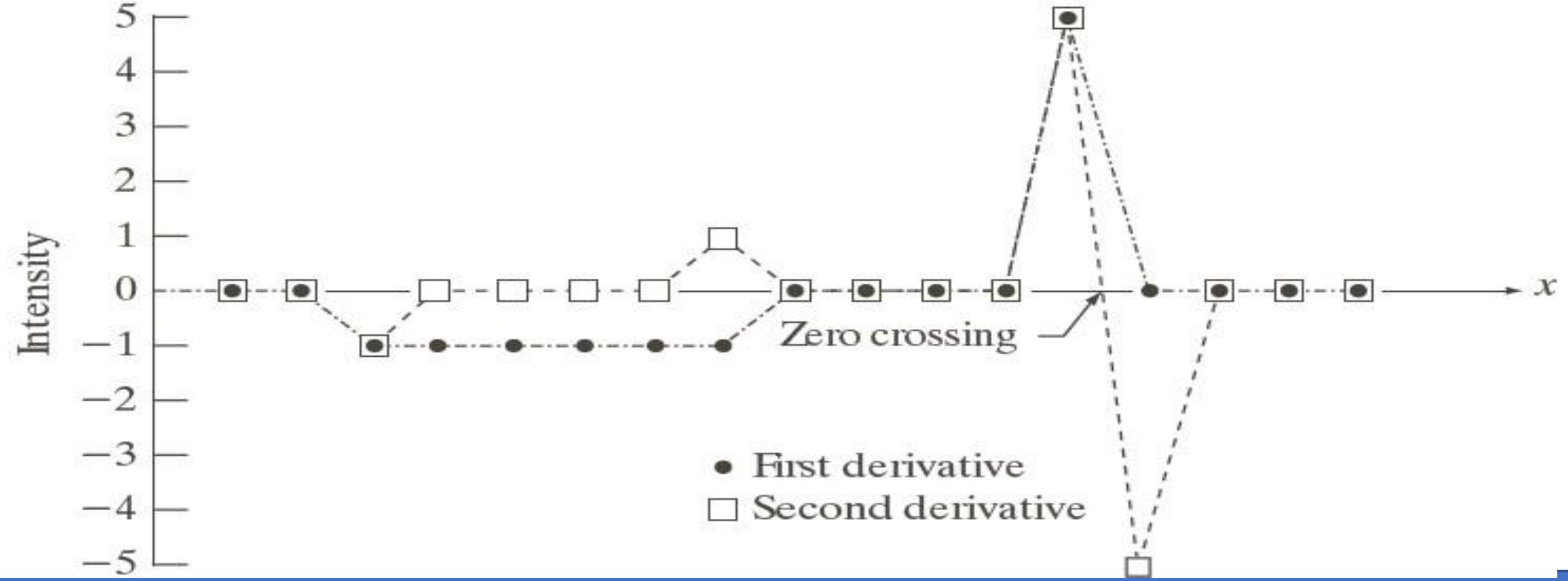
Foundation

- Definition for a first order derivative (1) must be zero in areas of constant intensity (2) must be nonzero at the onset of an intensity step or ramp and (3) must be nonzero along ramps.
- For a second order derivatives (1) must be zero in constant areas (2) must be nonzero at the onset and (3) must be zero along ramps of constant slope.
- First order derivative of a one dimensional function f(x) is the difference of f(x+1) – f(x).
- Second order = f(x+1) + f(x-1) -2f(x)



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	

$\rightarrow x$



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Second Derivatives-The Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y).$$

Second Derivatives - The Laplacian

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

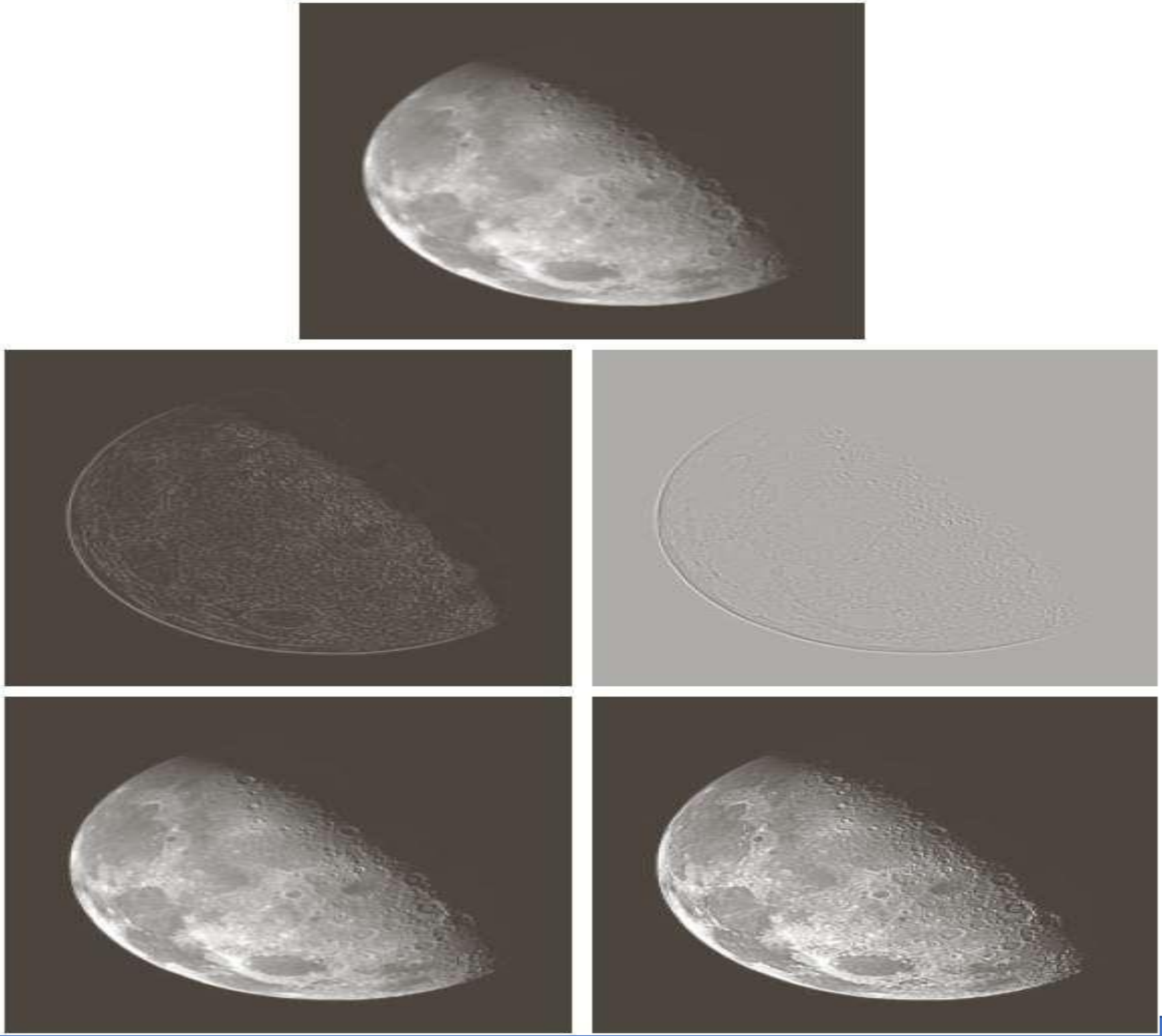
−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Second Derivatives-The Laplacian

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases}$$



	a	
b	c	
d	e	

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Unsharp Masking and High boost Filtering

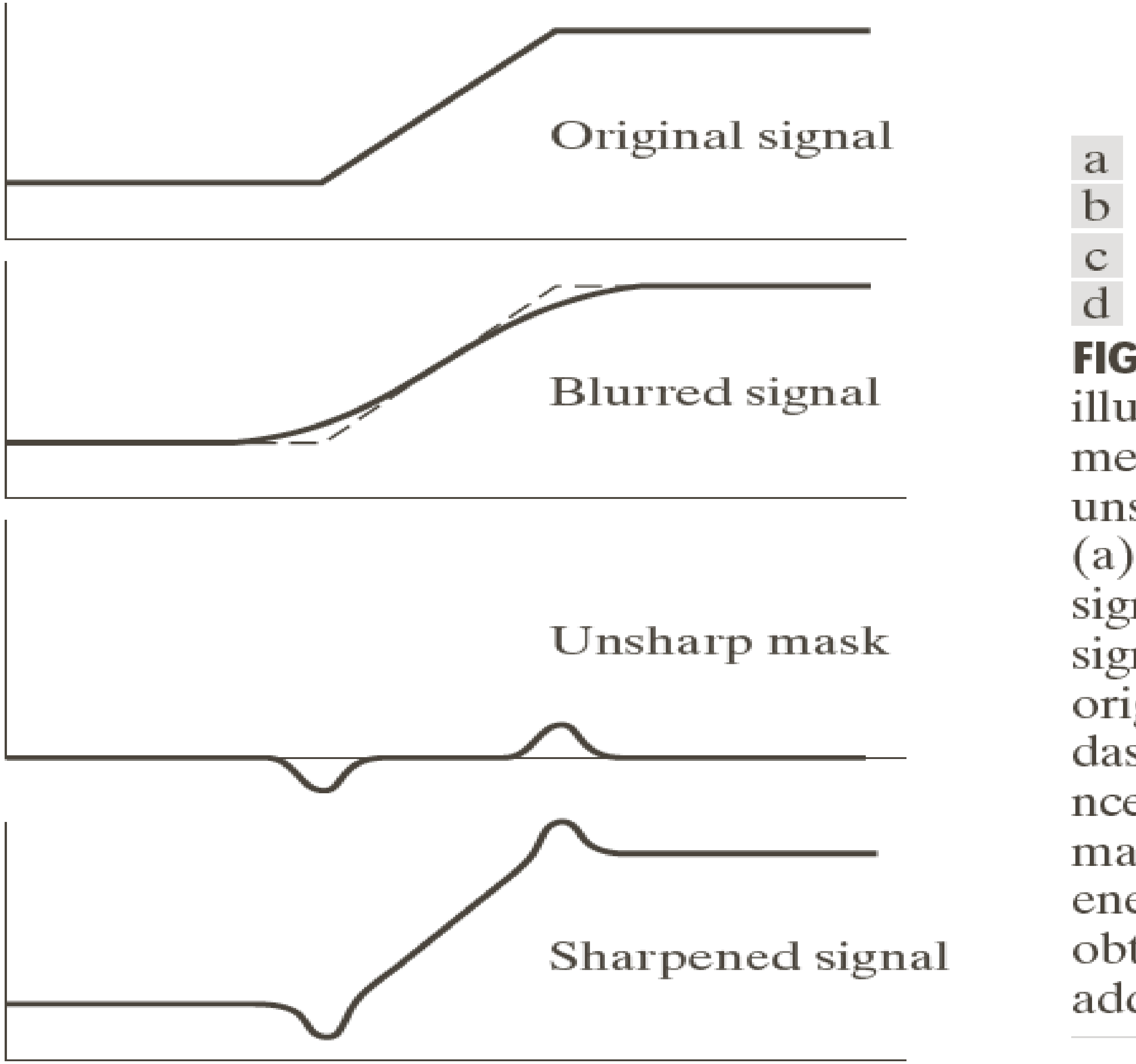


FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Unsharp Masking and High boost Filtering

- Unsharp Masking
  - Read Original Image f(x,y)
  - Blurred original image f'(x,y)
  - Mask = f(x,y) – f'(x,y)
  - g(x,y) = f(x,y) + Mask
- High Boost Filtering
  - Read Original Image f(x,y)
  - Blurred original image f'(x,y)
  - Mask = f(x,y) – f'(x,y)
  - g(x,y) = f(x,y) + k*Mask,   where k>1

Unsharp Masking and High boost Filtering

a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

First Derivative – The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\begin{aligned} \nabla f &= \text{mag}(\nabla \mathbf{f}) \\ &= [G_x^2 + G_y^2]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \end{aligned}$$

$$\nabla f \approx |G_x| + |G_y|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

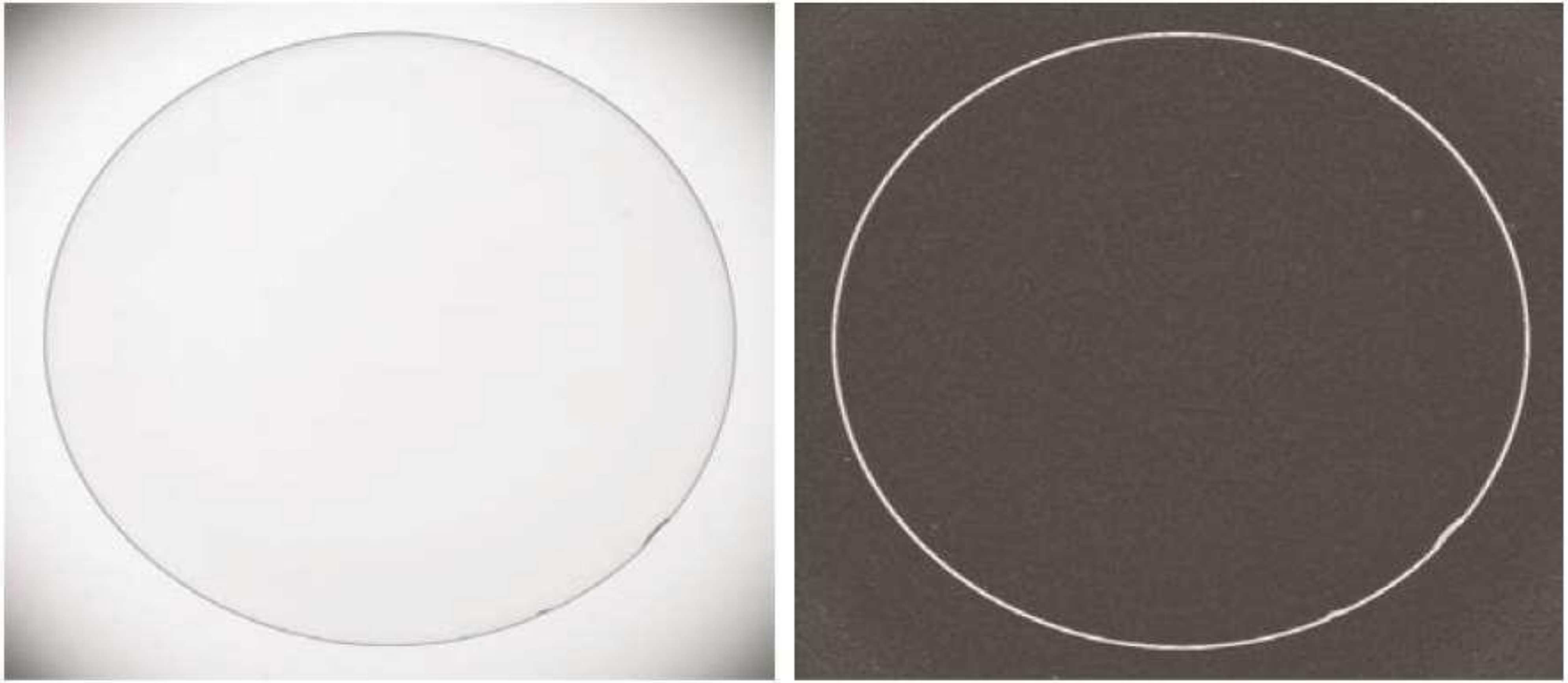
0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

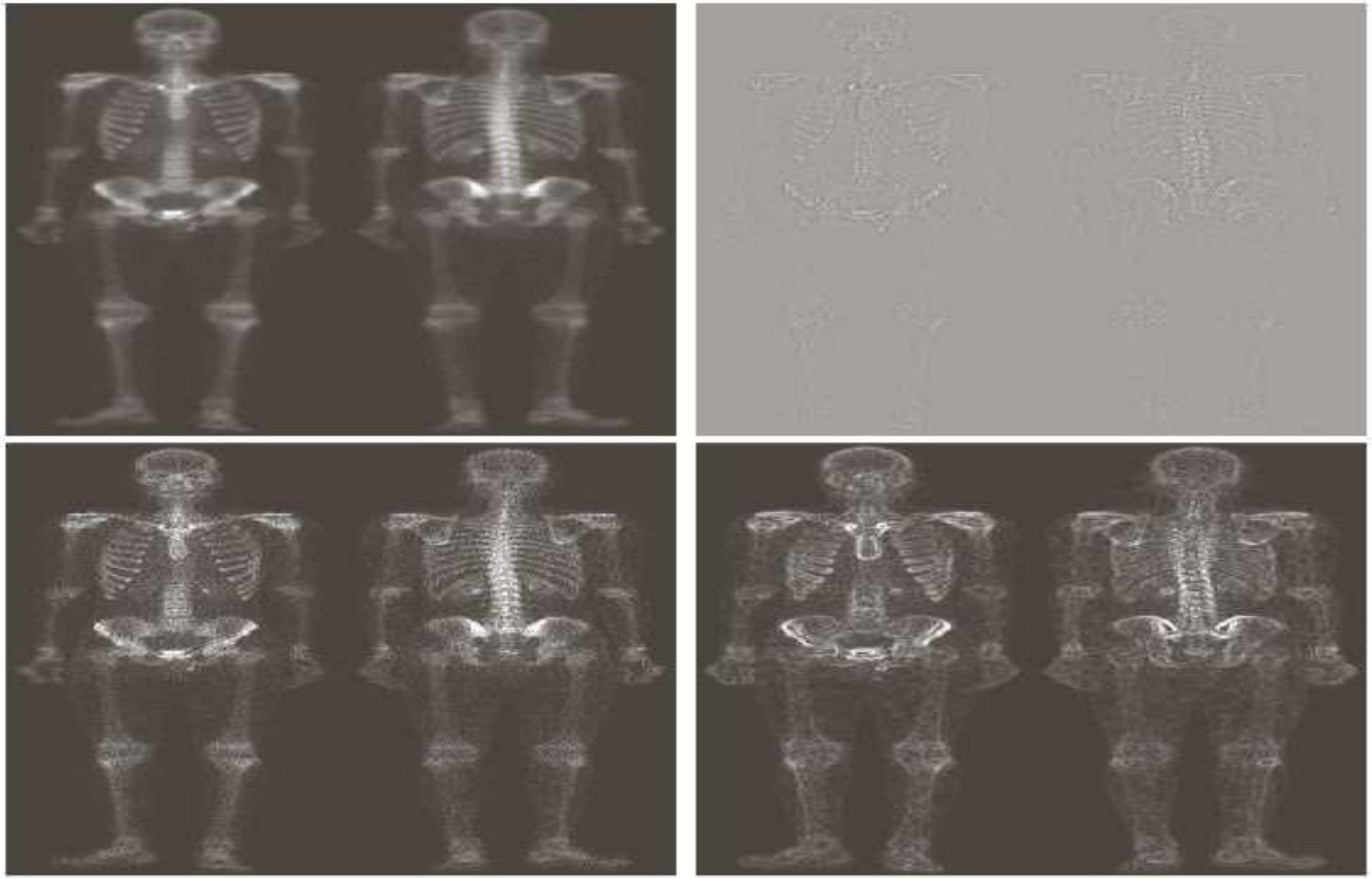
FIGURE 3.41 A 3×3 region of an image (the *z*s are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.



a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

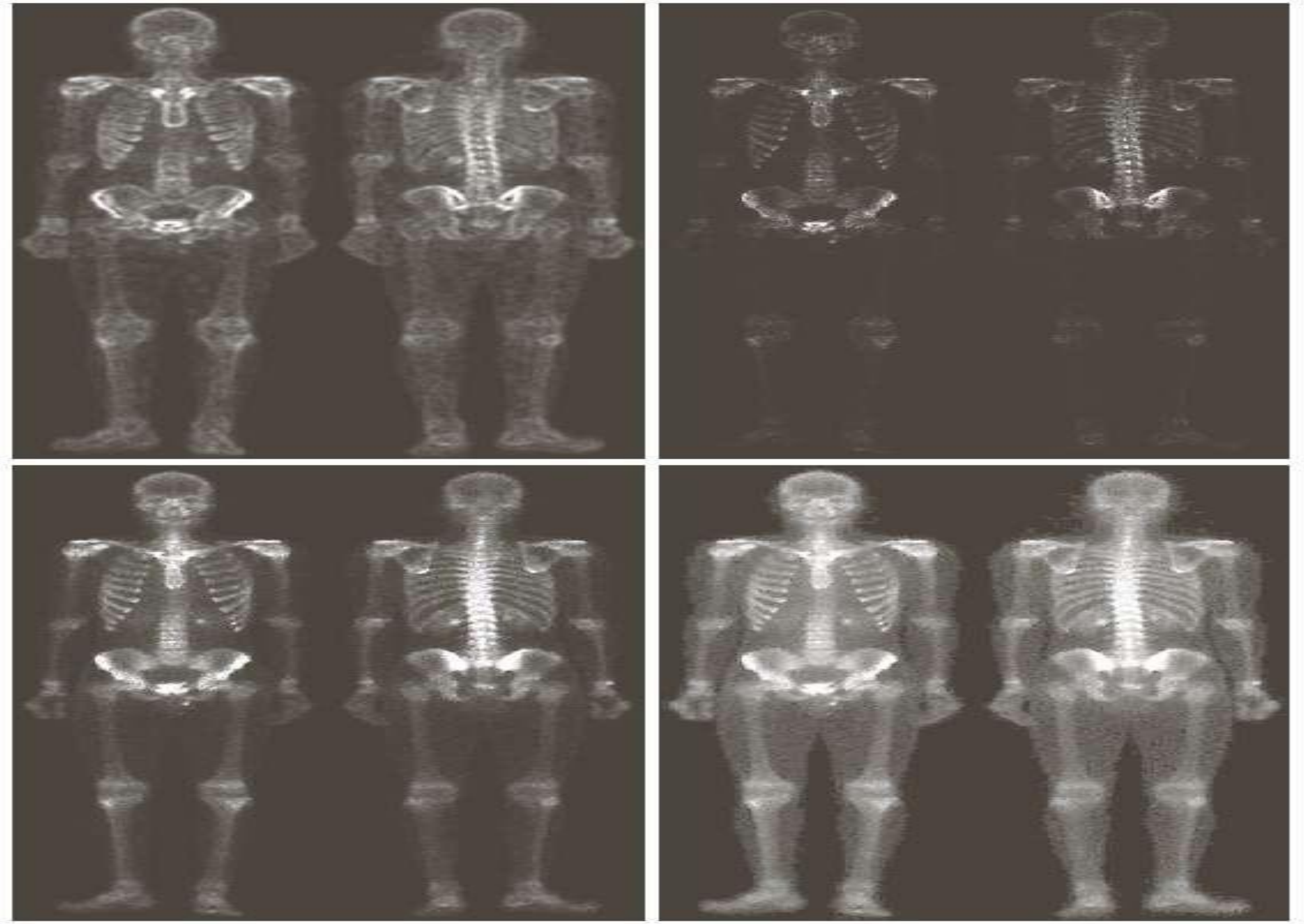
Combining Spatial Enhancement Methods



a	b
c	d

FIGURE 3.43 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Combining Spatial Enhancement Methods



e	f
g	h

FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

End of Lecture

Chapter 3 of Digital Image Processing by R.C. Gonzalez and R.E. Woods